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Wealth distribution and Pareto's law in the Hungarian medieval society

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Abstract

The distribution of wealth in the medieval Hungarian aristocratic society is studied and reported. Assuming the wealth of a noble family to be directly related to the size and agricultural potential of the owned land, we take the number of owned serf families as a measure of the respective wealth. Our data analysis reveals the power-law nature of this wealth distribution, confirming the validity of the Pareto law for this society. Since, in the feudal society, land was not commonly traded, our targeted system can be considered as an experimental realization of the no-trade limit of wealth-distribution models. The obtained Pareto exponent ($\alpha = 0.92$–0.95) close to 1, is in agreement with the prediction of such models.

Keywords: Wealth distribution; Econophysics; Pareto's Law

1. Introduction

At the end of the XIX century the economist Vilfredo Pareto [1] discovered a universal law regarding the wealth/income distribution in societies. His measurement results on several European countries, kingdoms and cities for the XV–XIX centuries revealed that the cumulative distribution of income (the probability that the income of an individual is greater than a given value) exhibits a universal functional form. Pareto found that in the region containing the richest part of the population, generally less than 5% of the individuals, this distribution is well described by a power-law (see for example Ref. [2] for a review). The exponent of this power-law is denoted by α and named Pareto index. In the limit of low and medium wealth, the shape of the cumulative distribution is fitted by either an exponential or a log-normal function.

The power-law revealed by Pareto has been confirmed by many recent studies on the economy of several corners of the world. The presently available data is coming from so apart as Australia [3], Japan [4,5], the US [6], continental Europe [7,8], India [9] or the UK [10,11]. The data is also spanning so long in time as ancient Egypt [12], Renaissance Europe [13] or the XX century Japan [14]. Since it is difficult to measure *wealth*, most

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of the available data comes from tax declarations of individual *income*. Empirical results are also available, which are based on a direct estimate of the wealth of individuals or institutions. The area of the houses in ancient Egypt [12], the inheritance taxation or the capital transfer taxes [15,11], the size/wealth of firms [7] or wealth rankings provided by some popular magazines [9,16] are examples of such studies. While wealth and income are related, one has to note that this relation is not a simple proportionality. The distribution of wealth is usually broader than the distribution of income, or equivalently, the Pareto index for wealth distribution is smaller than the corresponding one for income (see Ref. [9] as an example). In the present paper, we present and discuss empirical studies of wealth distribution in a medieval society—the Hungarian aristocratic society around the year 1550. In those times, the wealth of a nobleman was directly related to the size and agriculture potential of the lands he owned. To quantify this wealth, we take the number of owned serf (villein) families—a measure generally used by historians and for which well documented data exist. In a feudal society, land was not commonly traded. Moreover, in the Hungarian society, the family land was not divided among the children nor given as dowry—almost everything was inherited by the eldest son. The case under study offers thus a somehow idealized example of a system without a relevant wealth-exchange mechanism and may be taken as an experimental realization of the *no-trade* limit of current wealth-distribution models. Our results also give further evidence for the universal nature of Pareto's law.

2. Statistical physics approach to Pareto's law

Typically, the presence of power-law distributions is a hint for the complexity underlying a system, and a challenge for statistical physicists to model and study the problem. This is why Pareto law is one of the main problems studied in Econophysics. Since the value found by Pareto for the scaling exponent was around 1.5, Pareto law is sometimes related to a generalized form of Zipf's law [16] and referred to as Pareto–Zipf law. According to Zipf's law, many natural and social phenomena (distribution of words frequency in a text, population of cities, water level of rivers, users of web sites, strength of earthquakes, income of companies, etc.) are characterized by a cumulative distribution function with a power-law tail with a scaling exponent close to 1. It is, however, important to notice that in contrast to what happens with most exponents in statistical physics, the Pareto index α, may change from one society to another, and for the same society can also change in time depending on the economical circumstances [5,14]. The measured values of α for the individuals income distribution span a quite broad interval, typically in the 1.5–2.8 range; studies focusing on the wealth distribution show, however, smaller Pareto index values, usually in the 0.8–1.5 interval [17]. This large variation of α indicates the absence of universal scaling in this problem—a feature which models designed to describe the wealth or income distribution in societies should be able to reproduce.

Models for wealth distribution are defined by a group of agents, usually placed on a lattice, that interchange money following pre-established rules. In most cases the system will eventually reach a stationary state where some quantities, for instance the cumulative distribution of wealth $P_>(w)$, may be measured. Following these ideas, Bouchaud and Mézard [18] and Solomon and Richmond [19,20] separately proposed a very general model for wealth distribution. This model is based on the existence of multiplicative fluctuations acting on each agent's wealth plus a mechanism for wealth exchange among agents, which depends linearly on the agents' wealth.In a mean-field scenario (interactions of strength J among all the agents) the model predicts that α should increase linearly with J, with $\alpha = 1$ in the case of independent agents ($J = 0$). Similar conclusions were obtained for other interaction topologies and for a nonlinear version of the model [21].

Another large category of models recently considered are random asset exchange models (see Ref. [22] for a review). In these models, pairs of randomly chosen agents exchange part of their money while saving the remaining fraction. Trade (wealth exchange) is thus a crucial ingredient in these models. For randomly distributed (quenched) saving factors, a Pareto-type wealth distribution with $\alpha = 1$ is found [23,24]. Variants of this model with asymmetric exchanges (with respect to wealth) can yield $\alpha < 1$, by tuning the asymmetry parameter or its distribution [25]. These models are thus able to explain different Pareto index values by appropriately choosing the free parameters in the wealth-exchange rule.

From the modeling efforts made by the statistical physics community, one can conclude that the emergence of Pareto law can be explained from many different approaches. Reproducing the power-law like wealth distribution and reasonable values for the Pareto index does not seem to be a problem. Much more debate is on the relevance of these models to real social systems and on the practical interpretation of the model parameters (see Ref. [26] for a critical review).

3. Wealth distribution measurements in the Hungarian medieval society

To our knowledge, there is no available data concerning the wealth distribution and the Pareto law for the Central-Eastern European aristocratic medieval societies. A flourishing economic life, barter and wealth exchange developed very slowly in this part of Europe. A centralized and documented taxation system was also introduced relatively late. Furthermore, the wealth of the aristocratic families was attached to something which was not commonly traded: the land. During this ages, land was inherited only by the eldest son, and was not given as a dowry. Of course, this undeveloped economic life makes it hard to collect uniform, relevant and large-scale data on this society. However, once such data is obtained, it could be of decisive importance, since such a society represents an idealized case, where the agents are acting roughly independently. This simplified economic system provides thus an excellent framework to test, in a trivial case, the prediction of some wealth distribution models.

The first centralized data for 47 districts of the medieval Hungary (10 of them under Turkish occupation) is dated from 1550. The data, in a rough format, is available in a recent book [27] dealing with property relations of the XVI century Hungary. For each district, the nobles and religious or city institutions are arranged alphabetically and their wealth is grouped in six categories: number of owned serf families and their lands, unused lands, poor people living on their land, new lands, servants and others. Using a method accepted by historians [28], the number of owned serf families is taken as the best measure of the wealth of a noble family, providing a good estimate of the size and agricultural potential of the owned lands.

After a careful analysis of all districts and summing up the wealth for those families that owned land in several districts, we obtained a data set that is usable for wealth distribution studies. We also imposed a lower cut-off value, chosen to be 10 serf families, and disregarded thus the low and medium wealth aristocrats. This cut-off is necessary since historians suggest that the database is not reliable in the low wealth ranges. With the above constraints, our final database had data for 1283 noble families and 116 religious or city institutions. Considering an average of five persons per family (a generally accepted value by the historians and sociologists specialized in the targeted medieval period) we obtain that our sample contains around 6400 people. This represents the top 8% of the estimated 80 000 aristocrats living in Hungary at that period, and 0.2–0.3% of the estimated total population (2.7 millions) of Hungary in 1550.

A convenient way to look for evidence of a power-law is a *rank/frequency plot* [16]. Ranking the considered families after their wealth (rank 1 for the richest) and plotting the rank as a function of wealth, gives the cumulative distribution function for wealth up to a proportionality constant (equal to $1/N_t$, where N_t is the total number of families in the considered society). On log–log scales, the power-law will appear as a straight line with a negative slope, which yields the Pareto index. Notice, however, that for historical reasons, the wealth-distribution results are presented on graphs with reversed axes in some works [9,16].

The results obtained from our data set are plotted in Fig. 1. The power-law scaling is nicely visible on two decades in the figure, thus confirming Pareto law with an exponent $\alpha = 0.92$. This value is comparable with the one obtained for the top Indian society ($\alpha = 0.82$ and 0.91) [9], the wealth distribution of firms in France ($\alpha = 0.84$) [7] or the wealth distribution of the richest Americans ($\alpha = 1.09$) [16].

The validity of the Pareto law and the value of the Pareto index will not change considerably (we get $\alpha = 0.95$) if we study only the wealth distribution of the 1283 aristocratic families and neglect from the database the 116 institutions. Studying, however, only the wealth distributions of the institutions will not give a Pareto-like tail at all, and on the log–log scale we will get a constantly decreasing slope (empty circles in Fig. 1). It is also observable from Fig. 1 that the Pareto law breaks down in the limit of the very rich families, where the wealth is bigger than 1000 serf families. This is presumably a finite-size effect and such results are observable in other databases too [5,9].

In order to have some information on the time evolution of the Pareto index as well, the wealth distribution of the Hungarian aristocratic families in the 1767–1773 period was also studied. For this period we had rough data [29] available only for 11 districts. The sample was much smaller than for the year 1550: we had only 531 families and 65 institutions with total wealth greater than one serf family. However, as a compensation for the

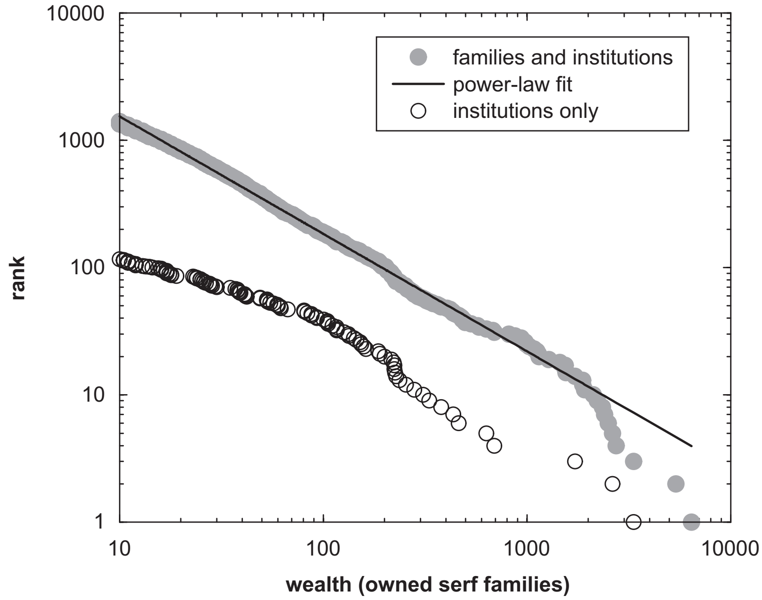

Fig. 1. The rank of the top 8% aristocrat families and institutions as a function of their estimated total wealth on a log–log scale. Measurement results for the Hungarian noble society in the year 1550. The total wealth of a family is estimated as the number of owned serf families. The power-law fit suggests a Pareto index $\alpha = 0.92$.

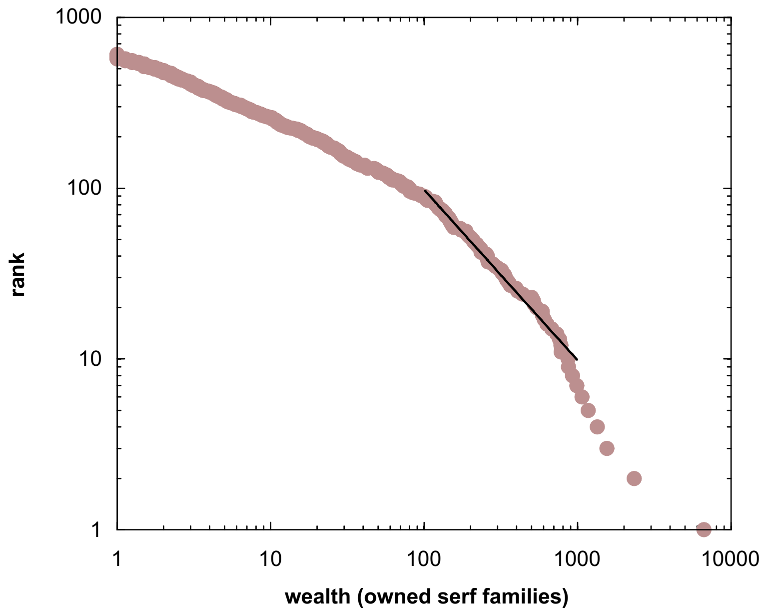

Fig. 2. The rank of noble families and institutions as a function of their estimated total wealth on a log–log scale. Data for the Hungarian noble society between the years 1767–1773. The power-law fit for the inner region suggests a Pareto index $\alpha = 0.99$.

smaller database, in this case the wealth of each family is given with three very clear markers: the owned number of serf families, the exact number of owned serfs and the total size of the owned land. To our great surprise, we obtained for each marker that the cumulative distribution function of wealth does not give the expected power-law behavior. Instead of a straight line with a negative slope, we found a constantly decreasing slope in the log–log plot of the rank as a function of wealth (Fig. 2). The fact that we used only data for 11 districts cannot be the reason for the breakdown of the Pareto scaling—we checked that, on the same 11 districts (and even smaller number of families), the 1550 data still gives the Pareto power-law tail. We believe the reason why the Pareto law is not valid for this database, is that a large wealthy part of the society is missing (similarly with the case of the year 1550 data, when only institutions are considered). Indeed, in the mid XVIII century in Hungary there were already many wealthy non-noble families of merchants, bankers, rich peasants, whose wealth exceeded the wealth of small or middle class aristocrats. This large category of relatively wealthy families were not landowners and had no serfs, so they are simply not present in the considered database. After our estimates, our wealth distribution data gives a reliable picture of the society only for the wealthier aristocrats, with wealth greater than 100 serf families. It is believed that the wealth of non-noble families that owned no land and serfs could not exceed this threshold. As observable, however, in the 1550 data, for wealth values larger than 1000 serf families finite-size effects are dominant, and the scaling breaks down. There is thus a very short wealth interval (one decade) where the data is trustful. Fitting a power-law on this interval leads to $\alpha = 0.99$. Although not too much confidence can be put on this value, one may use it to estimate the wealthy part of the society missing from our data (see Appendix). The results obtained in such a manner are reasonable ones.

4. Discussion and conclusions

Our study shows that the cumulative wealth distribution of the top Hungarian aristocratic families and institutions around the year 1550 exhibits a power-law shape with a Pareto index between 0.92 and 0.95. As mentioned in Section 2, a Pareto exponent equal to 1 is predicted both from a no-trade hypothesis and from exchange-based rules (with heterogeneous saving factors). The former assumption appears to us more adequate for modeling the social/economic situation under study, for the reasons explained above.

One can formulate the (independent agents) random multiplicative process for wealth evolution in a way which is similar to the reaction-like model used by Zanette and Manrubia [30] to explain the population size distribution of large cities [16,30]. Let us thus consider that the wealth of noble i, w_i, fluctuates in time due to several processes, either exogenous (wars, meteorological conditions affecting harvest sizes, ...) or endogenous (good or bad administration, gambling, ...). One can assume that

$$w_i(t+1) = \lambda_j w_i(t)$$

due to process j, which occurs with probability p_j (λ_j is a non-negative random variable). To prevent collapse to 0 in a finite system, one also has to add some noise term of zero average [30,31]. Assuming that the wealth stays constant on average, and that there are n such processes ($\sum_{j=1}^{n} p_j = 1$),

$$\langle \lambda \rangle = \sum_{j=1}^{n} \lambda_j p_j = 1.$$

One easily concludes (see Refs. [30,31]) that $p(w) \propto w^{-2}$ is a stationary solution for the probability density, which yields $\alpha = 1$ for the cumulative distribution. Thus considering uncorrelated multiplicative and additive stochastic fluctuations in wealth and conserved average wealth will lead to a Pareto index $\alpha = 1$. The value obtained in our study is quite close to this limit, so in principle we can admit that our results support the prediction of such models. We may not conclude, however, that the value $\alpha = 0.92$–0.95 found in our system is a clear indication of the absence of wealth trade. As discussed earlier, Pareto index values equal to 1 (and values even smaller than 1) can also be generated by random asset exchange models [23–25], where the wealth exchange is fundamental. Moreover these models are also capable of producing $\alpha < 1$ for some specific choices of the parameters.

In conclusion, we have shown that Pareto's law also holds for the top aristocrats in the Hungarian medieval society. If we admit that in this society the trade of wealth (land) was not significant, and that the obtained Pareto index value $\alpha = 0.92$–0.95 was measured with a considerable noise, we can interpret our results in the view of the no-trade limit of some recent wealth-exchange models.

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Appendix

Let us focus on wealth values greater than one serf family. If we assume that for wealthy families the cumulative distribution is scaling as $P_{\geqslant}(w) \propto w^{-1}$, the $p(w)$ wealth distribution density function has the form

$$p(w) = \frac{C}{w^2}. \tag{1}$$

The C constant can be determined by taking into account that we have 83 families and institutions with wealth between 100 and 1000 serfs

$$\int_{100}^{1000} \frac{C}{w^2}\,\mathrm{d}w \approx \frac{C}{100} = 83, \tag{2}$$

leading to $C = 8300$. From here it is immediate to estimate the N_1 total number of families or institutions in the targeted 11 districts of Hungary that have wealth greater than one serf family:

$$N_1 = \int_{1}^{W_0} \frac{C}{w^2}\,\mathrm{d}w \approx C = 8300 \tag{3}$$

(W_0 stands for the biggest reported wealth value, $W_0 = 6600$). This estimate can be confronted with the results of the census made between 1784 and 1787. For the studied 11 districts the total population was estimated to be around 1.6 million, with 45 thousand aristocrats and 33 thousand rich burgers. Taking again five members per family, our database shows that only around $531 \times 5 = 2655$ aristocrats had wealth bigger than the considered one serf family limit. This value is only 6% of the total estimated aristocrats in the society. Most of the aristocrats at that time had thus no considerable fortune except their noble title. The missing $8300 - 600 = 7700$ families corresponding to roughly $7700 \times 5 = 38\,500$ persons, should be mostly rich burghers or some richer peasants. This estimate is in reasonable agreement with the census from 1784 to 1787.

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