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Stochastic resonance in 3D Ising ferromagnets

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Abstract

Finite 3D Ising ferromagnets are studied in periodic magnetic fields by computer simulations, considering the classical heat-bath dynamics. The phenomenon of stochastic resonance is found. The characteristic peak obtained for the correlation function between the external oscillating magnetic field and magnetization versus the temperature of the system is computed for various external fields and lattice sizes.

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1. Introduction

It is well known [1–4], that periodically modulated bistable systems in the presence of noise exhibit the phenomenon of stochastic resonance (SR). The basis of this phenomenon is that the correlation σ between the modulation signal and the response of the system presents an extremum for a given noise intensity.

Evidence for SR was found in analog simulations with proper electronic circuits [3,5,6], laser systems operating in multistable conditions [7], electron paramagnetic resonance [8,9], in a free standing magnetoelastic ribbon [10] and in globally coupled two-state systems [11,12]. Theoretical aspects of the problem were reviewed in Ref. [4].

Considering a special and practically important case of coupled two-state systems, we recently reported on the possibility of obtaining SR in finite two-dimensional Ising systems [13]. In Ref. [13], in contrast with all other earlier works we did not consider any external stochastic force, only the thermal fluctuations in the system. The problem was studied by computer simulations considering a heat-bath dynamics.

The present paper intends to complete the earlier one [13], studying the important three-dimensional (3D) case.

2. The problem

To detect SR we need a system in a double well potential, governed by a stochastic force. In the meantime the two minima of the double well potential must be modulated periodically and in antiphase. One can immediately recognize that ferromagnetic Ising systems in oscillating magnetic fields satisfy all these conditions:

– At zero thermodynamic temperature the free-energy versus magnetization curve has the double well form;

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- An external periodic magnetic field $B(t)$ modulates the two minima in antiphase;

- The effect of a positive temperature can be viewed as a stochastic driving force;

- The magnetization as a function of time ($M(t) = \sum_i S_i^z$) can be considered as the response function of the system.

Due to the fact that in the proposed system the noise intensity (thermal fluctuations) is temperature dependent, the characteristic maximum for SR must be detected at a resonance temperature (T_r) in the plot of the correlation σ ($\sigma = |\langle B(t)M(t)\rangle|$) versus the temperature (T).

The Hamiltonian of our problem is

$$H = -J\sum_{i,j} S_i^z S_j^z + \mu_B B(t)\sum_i S_i^z, \tag{1}$$

where the sum is over all nearest neighbours, $S_i^z = \pm 1$, μ_B is the Bohr magneton and $B(t)$ the external magnetic field. We consider $B(t)$ in a harmonic form,

$$B(t) = A\sin(2\pi t/P). \tag{2}$$

We mention that recently 3D Ising systems in oscillating magnetic fields have already been considered by computer simulations [14]. However in Ref. [14] the authors study the hysteretic response of the system and no evidence for SR is discussed.

We will study the proposed system (1) from the viewpoint of SR.

3. The computer simulation method

To study the time evolution of the proposed system (1) first we considered a computer simulation to heat-bath dynamics. The time scale was chosen in a convenient way, setting the unit-time interval equal with the average characteristic time (τ) necessary for the flip of a spin. We have taken this time interval τ as constant, and thus independent of the temperature. Although this assumption is just a working hypothesis we expect it to give useful qualitative results. The spin flips were realized with the probabilities of the heat-bath dynamics, choosing the spins randomly at each moment.

The simulations were performed on cubic lattices with $W = N \times N \times N$ spins, considering a value of N up to 50. One simulation step was defined as W trials of changing spin orientations, and corresponds to a time interval τ. The period (P) of the oscillating magnetic field will be also given in these τ units. The amplitude A of the magnetic field is considered already multiplied by μ_B/k, and thus will have the dimensionality of the temperature (k is the Boltzmann constant). The temperature will be considered in arbitrary units, and the critical temperature of the infinite system ($T_c \approx 4.444J/k$) will be set to 100 units. Starting the system from a random configuration we considered 10000 simulation steps to approach the dynamic equilibrium. The correlation function between the driving field $B(t)$ and the magnetic response $M(t)$,

$$\sigma = |\langle B(t)M(t)\rangle| = \left|\frac{1}{n}\sum_{i=1}^{n} B(t_i)M(t_i)\right|, \tag{3}$$

was studied after this during 10000 extra iterations. (The averaging in (3) is as a function of time, and $t_i = \tau i$.)

The correlation (σ) was studied as a function of (i) the temperature (T), (ii) the lattice size (N), (iii) the amplitude of the magnetic field (A), and (iv) the period of the oscillating magnetic field (P).

4. Results

Our simulation results are summarized in Figs. 1–4.

In Fig. 1 we present a characteristic result for the shape of the σ versus T curve. One will observe that in accordance with the predicted phenomenon of SR, at a given T_r, σ presents a maximum. The tail of this resonance peak is nicely described by a power law (bottom). Analysing the scaling exponents for different simulations (different P, A and N values) we conclude that it lies in the interval (-1.7)–(-2.35).

Fig. 2 presents the shape of the resonance peak for several values of the modulation amplitude A. Our simulations (Fig. 2) predict that the resonance temperature is almost independent of the modulation amplitude, exhibiting only a very slight variation as a function of it (i.e. for higher amplitudes T_r is shifted in the direction of smaller values). In contrast with this, the height of the peak depends sensitively on the values of the modulation amplitude.

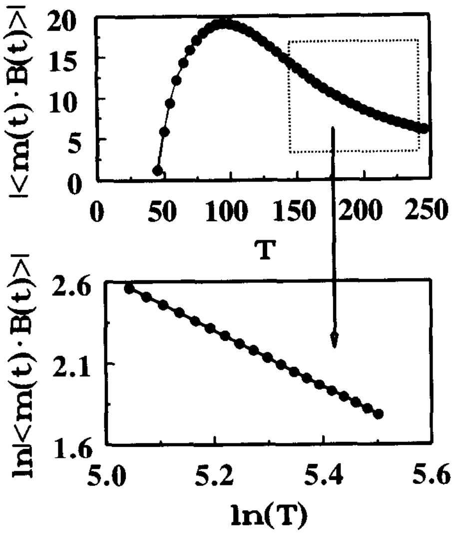

Fig. 1. Characteristic peak for the SR phenomenon obtained by our computer simulations. The bottom picture illustrates the $|\langle B(t)M(t)\rangle| = 11.181T^{-1.71}$ power law behaviour of the tail ($m = M/W$, $N = 20$, $A = 40$, $P = 100$ and $T_c = 100$).

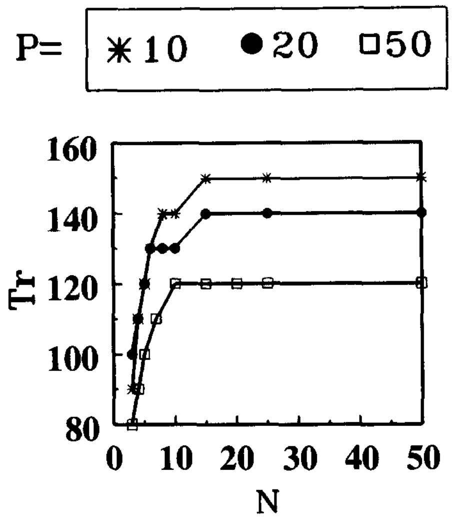

Fig. 3. Dependence of the resonance temperature (T_r) versus the lattice size (N). Results for three different modulation periods (P) are presented ($T_c = 100$ and $A = 10$).

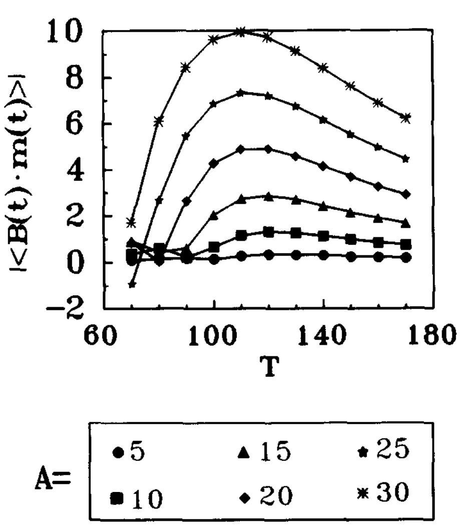

Fig. 2. Results for the shape of the SR peak considering several values of the modulation amplitude A ($m = M/W$, $N = 20$, $T_c = 100$ and $P = 50$).

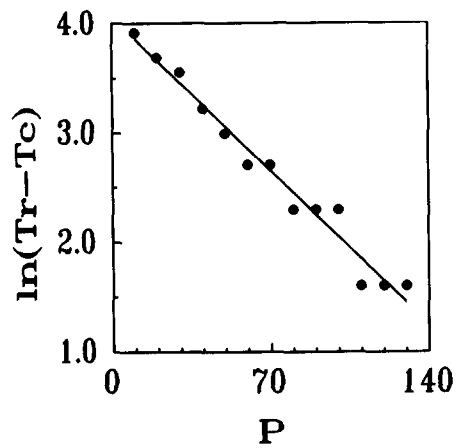

Fig. 4. Dependence of the resonance temperature $\ln(T_r - T_c)$ versus the modulation period (P). The best fit line indicates the exponential law $T_r - T_c = 4.061\,e^{-0.02P}$ ($A = 10$, $N = 20$).

As we concluded also in Ref. [13], for small lattices the T_r resonance temperature is strongly dependent on the lattice size (N), and in the limit of relatively big lattices ($N \approx 20$) tends to a constant limiting value. Our results are plotted in Fig. 3 for three different modulation periods P. From this figure we also learn that the resonance temperature T_r is dependent on the modulation period P. So we studied the dependence of T_r versus P. The results are given in Fig. 4. The simulation predicts that for long periods the T_r resonance temperature tends to the critical temperature T_c of the system. This dependence can be described by an exponential law,

$$T_r - T_c = K_1 \, e^{-K_2 P} . \quad (4)$$

We mention here that the step-like form of the results is due to the fact that to detect T_r the temperature was varied in steps of five units ($T_c = 100$ units).

5. Conclusions

The first conclusion would be that our computer simulations suggest that the phenomenon of SR should be detected when one studies finite ferromagnetic systems in oscillating magnetic fields. The characteristic peak of SR is obtained by studying the correlation $\sigma = |\langle B(t) M(t) \rangle|$ as a function of temperature. For a given resonance temperature T_r, this correlation σ exhibits a maximum.

Fixing the frequency, for small lattices ($W < 4000$) the T_r resonance temperature depends on the lattice sizes and tends to a limiting value for relatively large ($W > 4000$) lattices. The resonance temperature proved to be dependent also on the period of the magnetic field, and in the limit of large periods converges exponentially to the critical temperature of the system. Because in real experimental conditions we are in the very long period limit (the time unit in simulations is set by τ, the characteristic time for the flip of a spin), we expect T_r to be detected at T_c. We also concluded that the T_r resonance temperature is not significantly influenced by the amplitude A of the oscillating magnetic field, the value A determining mainly the height of the resonance peak.

The results obtained in the 3D case are qualitatively the same as our earlier results [13] for the 2D square lattice.

The obtained phenomenon could be interesting both from the theoretical viewpoint of statistical physics and for applications in magnetism. An experimental investigation of the problem would also be important.

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