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Synchronization of two-mode stochastic oscillators: a new model for rhythmic applause and much more

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Abstract

We model the collective clapping of spectators by globally coupled two-mode stochastic oscillators. All distinct experimentally observable clapping modes are successfully reproduced. Surprisingly, it is found that in an extended region of the parameter space the periodicity of the collective output is strongly enhanced by the considered coupling. The model offers a realistic way to generate periodic dynamics by coupling largely stochastic units.

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1. Introduction

Many physical, biological or sociological systems composed of a large number of oscillating and coupled units presents the phenomenon of synchronization [1]. Some well-known examples are the synchronization of pendulum-clocks hanging on a wall as first described by Huygens, arrays of bistable oscillators [2], flashing of fireflies [3], neurons [4], pacemaker cells in the heart [5], chirping of crickets [6], menstrual cycle of women living together [7] and the recently discussed phenomenon of rhythmic applause [8,9]. In order to construct a realistic model for the above phenomena

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one has to take into account the stochasticity in the period of the oscillators, their different natural frequencies and sometimes also their different operational modes. Moreover, the coupling has to be considered also in a realistic manner. The question then whether synchronization in such system is possible or not becomes interesting and non-trivial. In the limit of large number of units statistical physics models and methods can become useful. Continuing our previous studies [8,9], in the present paper we give an alternative description and model for the phenomenon of rhythmic applause.

In our earlier studies we approached collective human clapping by using the simple Kuramoto model [10], which is well-known in statistical physics. Within this approach the clapping spectators are modeled by phase-coupled rotators. The rotators have different natural frequencies and they are globally and uniformly coupled. Following our experimental investigations, we proposed that the rotators can operate either in low or a high frequency mode. Applying then the results of the Kuramoto model, we were able to qualitatively explain the characteristic features and intermediate regimes in collective clapping. Particularly, we proved that in order to achieve synchronization the audience has to clap with the low frequency mode. We also showed that the fascinating and peculiar interplay between synchronized and unsynchronized regimes in rhythmic applause is a consequence of a frustration in the system. The audience has two conflicting desire, first to produce large noise intensity, and secondly to achieve synchronization. While large noise intensity can be achieved by clapping in the fast mode, seemingly synchronization can be obtained only in the low frequency clapping mode. We argued that this conflicting desire is responsible for the characteristic dynamics of rhythmic applause.

The description based on the Kuramoto model is of course a very crude approximation to reality. First, it does not take into account the stochasticity of the individual oscillators, and secondly the global phase coupling is unrealistic. Coupling should be realized either through the global output of the system, or by a hierarchical consideration of the neighbors. Modeling the clapping person by a rotator is also a rough approximation and neglects the different phases in a clapping cycle. The model we describe in the present study has the aim to overcome the shortages of the previous description. Beside the immediate interest in explaining this fascinating social behavior, we show that this new model is useful for explaining other puzzling collective biological phenomenon as well, or could be useful in designing self-correcting clever systems with highly stable periodic output.

2. Facts about human clapping

Previous experiment revealed interesting properties of human clapping.

Regarding the clapping of *one isolated individual*, Musha et al. [11] showed the stochastic nature of the clapping period. He also argued that clapping has features resembling $1/f$ noise. Controlled experiments performed by us on high-school students [8,9] supports the results of Musha et al. and also revealed the existence of two distinct clapping modes. One mode is characterized by a lower frequency and corresponds to the natural clapping of the individuals. The second mode is an "excited", fast clapping mode, with a roughly doubled frequency.

For the *collective clapping* of an audience one can distinguish four qualitatively different modes. A first mode is the weak "non-enthusiastic" clapping, encountered after bad or average performances. Usually there is no synchronization in this mode, and the average output intensity is low. A second possible mode is the thunderous clapping, characteristic for an excited audience. The audience rewards exceptional performances by this clapping mode. It is characterized by high output intensity, no synchronization and fast mode clapping of the spectators. Another clapping mode which appears also after exceptional performances is the so-called *rhythmic applause*. Rhythmic applause is characterized by synchronized regimes and can appear in two forms. The most common form has a frequency in the neighborhood of the average slow clapping frequency of the spectators. A second form of rhythmic applause is characterized by synchronized high frequency clapping. This later form is, however, very unstable and can be sustained only seldom. Rhythmic applause has a complex dynamics, synchronized clapping alternating with the thunderous clapping mode. Audio and video samples for rhythmic applause can be downloaded from the web-page dedicated to our previous studies [12].

In *conclusion* experiments on individuals revealed both the stochastic nature of clapping and the two possible clapping modes. Collective clapping of an audience can lead to four qualitatively different modes, distinguished by the intensity of the global clapping, synchronization, and frequency.

3. The model

The model introduced in this section describes in a realistic manner the stochasticity and pulse-like output of the clapping phenomenon, the two co-existing clapping modes, and considers a realistic coupling through the global output of the system. It does not take into account, however, the differences in the natural frequencies of the oscillators. Since the stochasticity of the oscillators introduces already a frequency fluctuation, we expect that our results will be qualitatively unmodified for a small non-zero standard deviation of the oscillators natural frequencies.

The model considers an ensemble of N oscillators. The cycle of each oscillator is composed of three parts denoted by A, B and C. The oscillators operate by following $A \to B \to C \to A \to B \to \cdots$ periodic route. The total period of the oscillator, T, is given then as $T = \tau_A + \tau_B + \tau_C$, where τ_A, τ_B and τ_C denotes the time interval spent in regime A, B and C, respectively. The stochastic part of the dynamics is A, and τ_A is thus a stochastic variable. This regime is intended to reproduce the stochastic reaction-time of an individual. Following the accepted models of neuron dynamics [13] we model this stochastic regime by an escape-dynamics of a stochastic field-driven particle from a potential valley. The time length τ_A is thus governed by the

$$P(\tau_A) = \frac{1}{\tau^*} \exp\left(-\frac{\tau_A}{\tau^*}\right) , \tag{1}$$

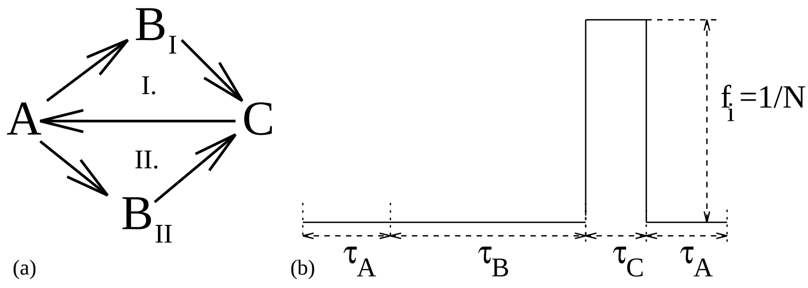

Fig. 1. (a) Dynamics of the stochastic oscillators, and the two possible modes. (b) Pulse-like output of the stochastic oscillator as a function of time.

distribution function. It is easy to realize, that the average value of τ_A is τ^*. Part B represents the τ_B deterministic period, the individuals would like to impose. Part C is also a deterministic regime, where the oscillator emits a constant $f_i = 1/N$ intensity pulse. The dynamics of the oscillators and it's output is sketched in Fig. 1. As shown in the figure, the two distinct clapping mode of an individual can be taken into account by allowing two different cycles for an oscillator. These two cycles are different by the time-length of the B regime. An oscillator can follow thus either mode I ($A \to B_I \to C \to A \to B_I \cdots$), or mode II ($A \to B_{II} \to C \to A \to B_{II} \cdots$). In order to reproduce realistically the two clapping modes we have chosen the lengths of the B_I and B_{II} regimes as observed experimentally: $\tau_{BII} \approx 2\tau_{BI}$.

Shifting between the two possible modes and the coupling of the oscillators is realized through their collective output. Let us assume that f^* is an average output intensity that the oscillators want to impose. The dynamics of the ensemble is described then by the following rules:

(1) Each oscillator starts with a randomly selected mode and phase (A, B or C), and follows the usual dynamics in the selected mode.
(2) After completing part A, each oscillator compares the

$$f = \sum_{i=1}^{N} f_i \ , \tag{2}$$

total output of the system with f^*.
(3) If $f < f^*$ the oscillator will follow mode I with shorter period, in order to increase the average output in the system. If $f > f^*$ the oscillator follows mode II with larger average period, in order to decrease the global output.
(4) The oscillators continue indefinitely their dynamics following rules (2) and (3).

The above dynamics is designed to keep the average output in the neighborhood of the desired f^* value. The coupling is thus realized through the global output of the system. It is important to notice that the tendency and aim for synchronization is not a priori introduced in the dynamics. In our opinion this model describes in a realistic manner the clapping individuals and their collective desire, which is to produce a given intensity clapping, appropriate to the quality of the performance.

4. Results

Let us fix the values of τ_C and τ_{BI} ($\tau_{BII} = 2\tau_{BI}$) and study the model on the $\tau^* - f^*$ parameter space.

For $f^* \leqslant 0$ all units operate in mode II, trying to decrease the average output of the system, which in this case is

$$\langle f \rangle_0 = \frac{\tau_C}{T_{II}} = \frac{\tau_C}{\tau^* + \tau_{BII} + \tau_C} \; . \tag{3}$$

This coupling is not effective, and after all the oscillators had switched to mode II they will just keep this mode. No synchronization can be achieved in this limit. Moreover, even if all oscillators start in mode II and in a synchronized manner, due to the stochasticity in their periods this synchronization is quickly lost. For $f^* = 1$ all units will tend to oscillate in mode I, trying to increase the average output, which in this case is

$$\langle f \rangle_1 = \frac{\tau_C}{T_I} = \frac{\tau_C}{\tau^* + \tau_{BI} + \tau_C} > \langle f \rangle_0 \; . \tag{4}$$

Again, the coupling is not effective and the oscillators cannot achieve synchronization. It is easy to realize that the above considerations should hold in the $f^* > \langle f \rangle_1$ limit, as well. For $0 < f^* < \langle f \rangle_1$ the dynamics is non-trivial and the oscillators continuously shift between the two possible modes. This regime can be studied however, only computationally. Fixing $\tau_C = 0.1$ and $\tau_{BI} = 0.4$, we have mapped the relevant $\tau^* - f^*$ parameter space by studying the global output after dynamical equilibrium was achieved. Four phases, distinguished by qualitatively different dynamics were revealed (Fig. 2e). For small f^* values we obtained a non-synchronized low intensity output (phase I). A global signal characteristic for this phase is plotted in Fig. 2a. By increasing the value of f^* a partially synchronized dynamics is observed (phase II). In this phase the global output of the oscillators show a periodic nature (Fig. 2b), with a frequency close to the slow mode of the individual oscillators. In a quite narrow $f^* - \tau^*$ parameter space a partially synchronized regime with a high intensity global output is observed (phase III). The frequency of this global output (Fig. 2c) is close to the high frequency mode of the individual oscillators. Finally, for high f^* values, again no synchronization can be achieved (phase IV) and the global output has a high average intensity, close to the value of $\langle f \rangle_1$. A global output characteristic for this regime is plotted in Fig. 2d. We conclude thus, that as a function of the τ^* and f^* parameters the system can exhibit four qualitatively different types of dynamics. Two of these is characterized by partial synchronization of the oscillators.

It is interesting and important to note that in the partially synchronized phase II the global output has a strong periodic nature. A first qualitative comparison with the signal given by one oscillator shows that the global output becomes much less stochastic

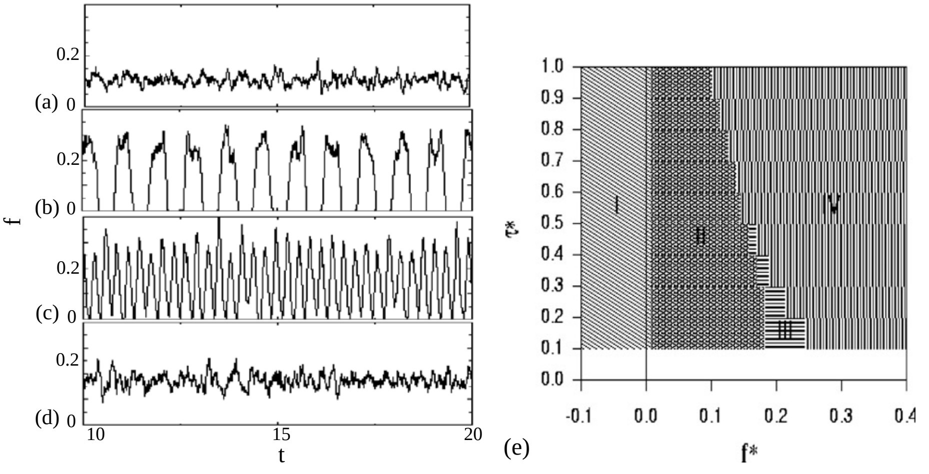

Fig. 2. (a)–(d) Global output of the system in the different phases, (a) phase I, (b) phase II, (c) phase III, (d) phase IV. (e) Phases in the $f^* - \tau^*$ parameter space.

than the output of a single oscillator following either mode I or mode II. This means that this setup is appropriate to enhance the periodicity of stochastic oscillators. In the rest of this section we investigate quantitatively this enhancement. In order to characterize numerically the enhancement in the periodicity, first we need to define a measure for it. Let us denote the investigated signal by $f(t)$. We can then define an error function, $\Delta(T)$, which characterizes how strongly the $f(t)$ signal is different from a periodic signal with period T

$$\Delta(T) = \frac{1}{2M} \lim_{x\to\infty} \frac{1}{x} \int_0^x |f(t) - f(t+T)| \, \mathrm{d}t \; , \tag{5}$$

where

$$M = \lim_{x\to\infty} \frac{1}{x} \int_0^x |f(t) - \langle f(t)\rangle| \, \mathrm{d}t,$$

$$\langle f(t)\rangle = \lim_{x\to\infty} \frac{1}{x} \int_0^x f(t) \, \mathrm{d}t \; . \tag{6}$$

The general shape of $\Delta(T)$ is sketched in Fig. 3.

For any $f(t)$ oscillating function, $\Delta(T)$ has an initially increasing tendency at small T values, after which for a $T = T_m$ period a Δ_m minimum is reached. One can then consider that T_m is the best approximation for the period of the $f(t)$ signal, and the level of periodicity of the signal is characterized by

$$p = \frac{1}{\Delta_m} \; . \tag{7}$$

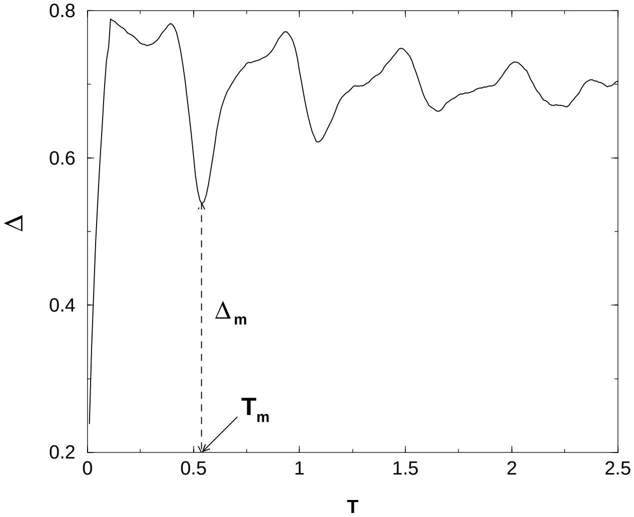

Fig. 3. A general shape of the $\Delta(T)$ function.

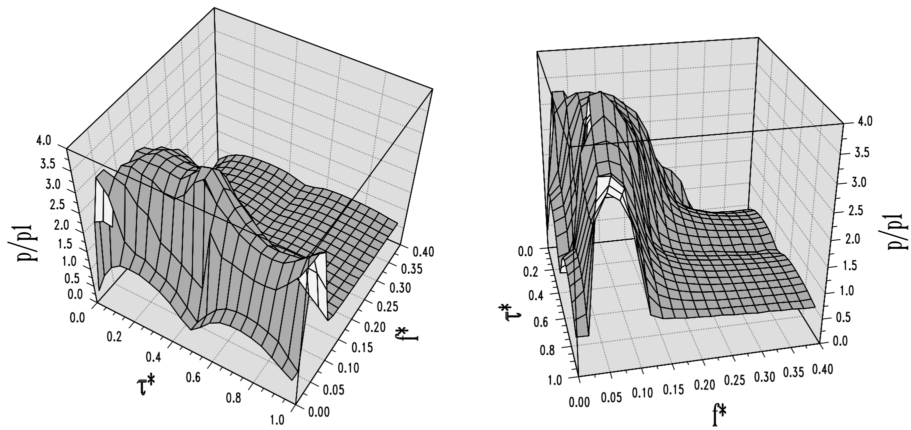

Fig. 4. Enhancement in the periodicity in the relevant $f^* - \tau^*$ phase-space. Two different views.

For a single oscillator the highest periodicity is obtained while operating in mode II, since in this slow mode the relative length of the τ_A stochastic time-interval is shorter. Let us denote this periodicity by p_1, and the periodicity of the global output simply by p. Then, the p/p_1 ratio will characterize the enhancement in the periodicity. We have studied this enhancement throughout the whole relevant $\tau^* - f^*$ parameter space. Results for $N = 200$ oscillators, $\tau_C = 0.1$ and $\tau_{BI} = 0.4$ are plotted in Fig. 4.

The data indicates that in a quite extended parameter range (corresponding mainly to phase II) the enhancement is strong. Moreover, increasing the number of oscillators

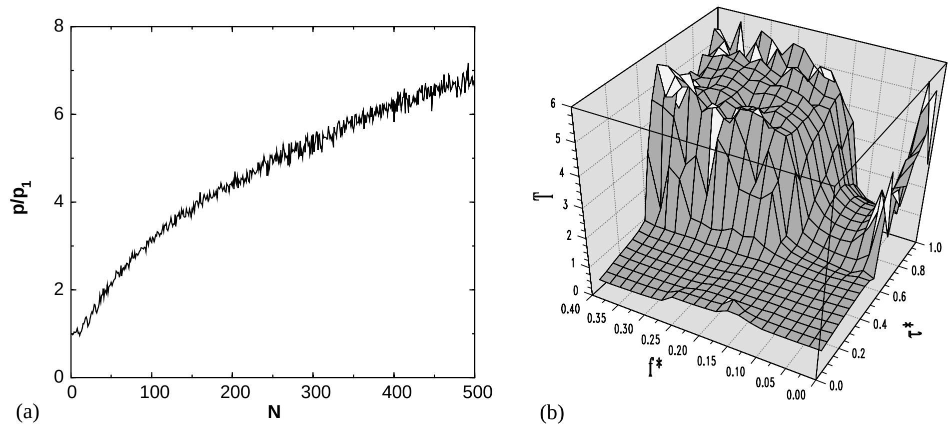

Fig. 5. (a) Enhancement in periodicity as a function of the number of oscillators ($\tau^* = 0.4$, $f^* = 0.1$, $\tau_{BI} = 0.4$ and $\tau_C = 0.1$). (b) The best T_M periods in the $f^* - \tau^*$ parameter space.

will further increase the periodicity of the global output. To illustrate this, we fixed $\tau^* = 0.4$ and $f^* = 0.1$ and investigate the enhancement as a function of the N number of oscillators. Results are summarized in Fig. 5a. From this result we conclude that the periodicity is monotonically increasing with the size of the system, although this increase is slower than a linear trend. Finally, we also mention here that the best T_m period obtained for the global signal (Fig. 5b) supports our previous statements about the four possible phases in the global dynamics of the system.

5. Discussion

The four qualitatively different dynamics obtained in our two-mode stochastic oscillator model reproduces the observed four distinct type of collective clapping modes. Phase I reproduces the non-enthusiastic clapping mode, characteristic after bad or average performances. In this case the spectators are satisfied with a low intensity average output. Phase II reproduces the synchronized regime of the common rhythmic applause, and phase III the fast-mode rhythmic applause. The parameter space where the fast-mode rhythmic applause appears is quite restricted and in accordance with the observations, our model predicts that this should be a quite rare phenomenon. Phase IV corresponds to the thunderous clapping mode of the excited audience, when high average output is desired. The present model suggests that in order to get rhythmic applause after an outstanding performance the average noise level of the thunderous clapping should be lowered. This is in fact what happens in reality. In order to get synchronized clapping the audience lowers detectably the average clapping output intensity and achieve synchronization in the low frequency clapping mode. Frustration due to the two conflicting desires: to get synchronization and high intensity clapping is responsible for the characteristic interplay of synchronized and thunderous clapping.

Our previous theory based on the Kuramoto model yield the same conclusion [8,9]. The ideal clapping mode of an excited audience would be of course to synchronize with high output intensity. According to the results of the present model this fast-mode synchronization can be achieved only under very restricted parameter values, and it is highly improbable for "untrained" audience. Seldom, especially in East-European countries, one can observe this synchronization mode, as well.

Apart from the interest in explaining the fascinating phenomenon of rhythmic applause, the present model could become useful also for understanding synchronization of other biological or sociological systems where two or more oscillating modes are present. As such examples we mention here the uni-cellular alga Gonyaulax polyedra in which circadian oscillators with two different periods coexist [14], or the thalamocortical relay neurons which can generate either spindle (7–14 Hz) or delta (0.5–4 Hz) oscillations [15].

From Fig. 4 it is also interesting to observe a stochastic resonance-type effect [16]. For fixed f^* values it seems that there is an optimal τ^* noise-level, where the maximal p/p_1 enhancement is achieved.

Finally, in our opinion the most important message from the present model is that periodicity enhancement can be obtained by coupling in a simple and realistic manner an ensemble of two-mode stochastic oscillators. One can thus design devices giving well-controlled periodic pulses (clocks) by using largely stochastic elements. Such biology-inspired technology would lead to clever, self-correcting clocks or bistable elements.

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