STOCHASTIC RESONANCE AT PHASE NOISE

K. LOERINCZ, G. BALAZSI[$], Z. GINGL, [+]L.B. KISS
JATE University, Dept. of Experimental Physics, Dóm tér 9, Szeged, H-6720 Hungary
[$] *Babes-Bolyai University of Cluj-Napoca, Kogalniceau street 1., Cluj-Napoca, 3400 Romania*
[+]*Uppsala University, Angstrom Lab, Box 534, Uppsala, S75121 Sweden*

A surprising new stochastic resonance phenomenon is reported. The particular set-up is a level-crossing detector with a supra-threshold sinusoidal excitation, and the noise is an additive band-limited white noise in the phase angle of the sinusoidal time-function. We observe stochastic resonance phenomenon in the first harmonic at the output power spectral density of the system. It means that, if the phase noise is not zero, there is an optimal strength of that phase noise where the signal to noise ratio reaches a local maximum. At the moment, there is no theory to explain this phenomenon.

1 Introduction

Stochastic resonance (SR) is a widely investigated phenomenon of statistical and solid state physics. SR can occurs in special nonlinear systems, when one can identify a periodic and noisy input excitation. SR means that the signal-to-noise ratio (SNR) at the output has a maximum as a function of the intensity of the input noise [1]. There are several real and many artificial systems, which can produce stochastic resonance [1], for example SQUIDS, noisy neural networks, lasers, level-crossing detectors [2-6], Schmitt-triggers and a lot of different mathematical models. First, SR was observed and explained in dynamical, two-state systems, however, later it was shown, that the real meaning of SR is different: SR is basically a level-crossing dynamical problem of the noisy signal [5]. Recently a promising possibility of increasing the output SNR over the input SNR in level-crossing detectors has been reported [5,6], highlighting the application possibilities of SR.

Previously, input signals used for stochastic resonators were usually additive, sometimes multiplicative, with the input signal. Here, we introduce a new type of stochastic resonance based on random phase modulation of the periodic input signal.

2 Model

The principle of the new stochastic resonator set-up can technically be visualised by a phase modulator followed by a level-crossing detector (LCD) [2], see Figure 1. The input of the phase modulator is a sine wave with frequency f_0 and the modulating input is fed by a band-limited white noise serving as the phase modulating quantity. For simplicity, the amplitude distribution of the phase noise has been chosen to be uniform. The phase modulator adds the noise to the phase of the input sine wave which can be expressed in the following way:

$$y(t) = A \sin[2\pi f_0 t + w(t)] \tag{1}$$

where w(t) represents the phase noise.

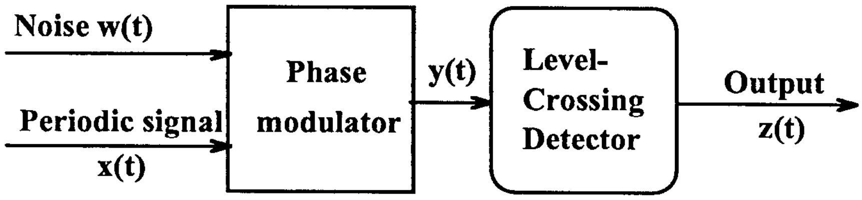

Figure 1: Visualisation of the set-up for stochastic resonance at phase-noise.

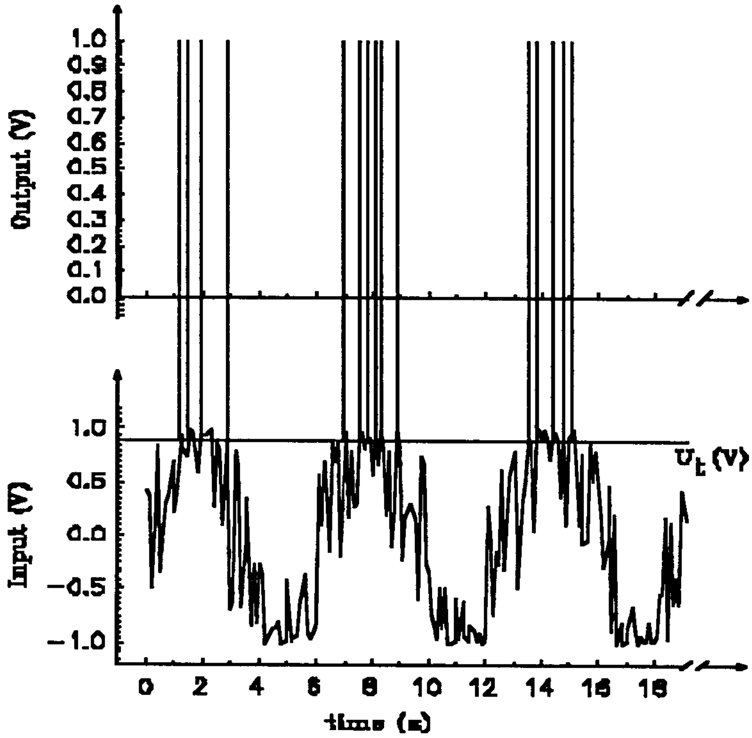

Figure 2: An example for the input and output amplitudes

The phase modulated sine wave gets into an LCD to produce the output signal. The LCD emits a pulse, whenever its input value is crossing the threshold value U_t upwards, and the LCD output is zero otherwise. Figure 2 shows a typical input and the corresponding output signal of the system.

3 Results

We used numerical simulations to investigate the behaviour of the phase modulator based SR. A (high-performance) uniform random number generator was used to represent the input noise. These numbers were used to generate samples of length of 16000. The power spectrum was calculated using FFT and by averaging 1000 samples. An example for the output spectra can be observed on Figure 3.

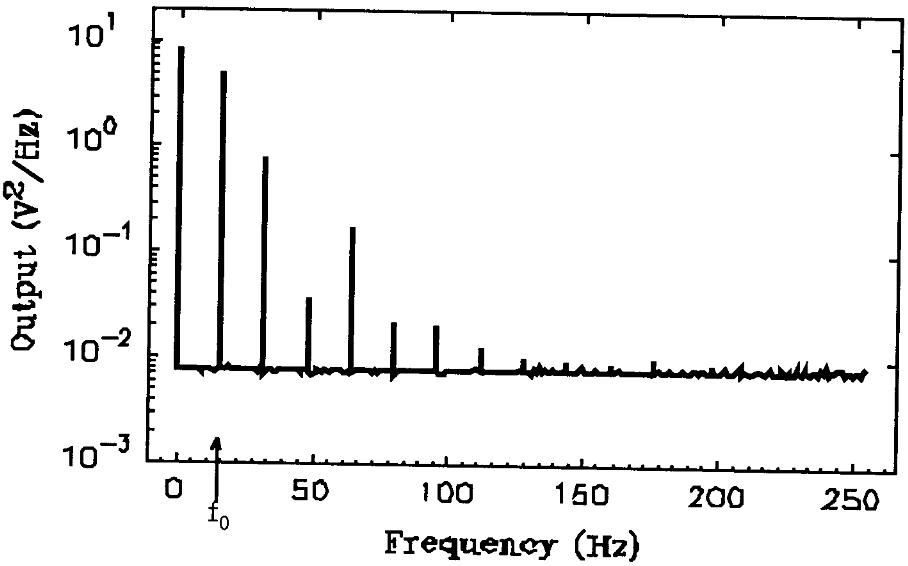

Figure 3:. An example for the output power density spectrum

The amplitude of the sine wave was fixed to unity, and the threshold and noise amplitude were varied. The output signal strength and the SNR values were calculated and plotted as the function of the threshold of the LCD and RMS value of the input noise. This dependence is illustrated on Figure 4. The existence of strong stochastic resonance is very clear from the behaviour of both quantities.

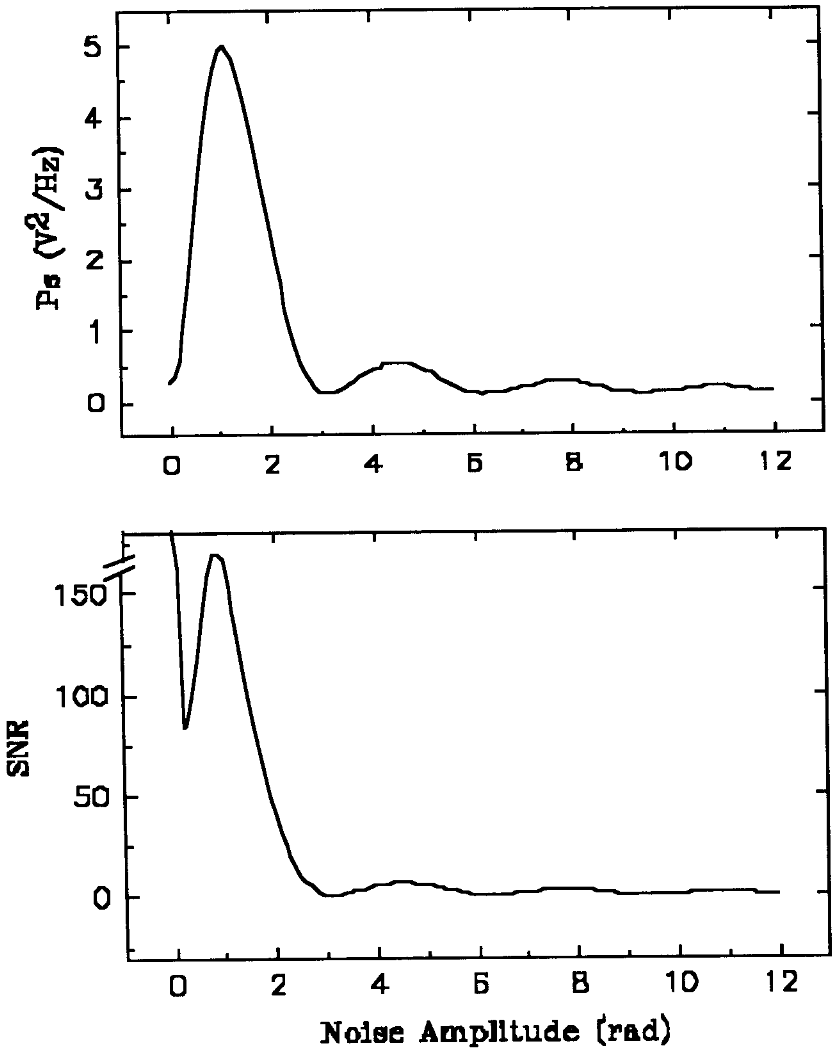

Figure 4: The output mean-square signal amplitude P_S and SNR versus the phase noise amplitude

It is important to note that to get any signal at the output, the system has to work in the *supra-threshold* limit:

$$-A < U_t < A \quad , \tag{2}$$

otherwise the input signal never crosses the threshold. On Figure 5, we can see the output spectrum when the threshold level varies between its limits. This behaviour is completely different from the previously considered SR systems additive noise.

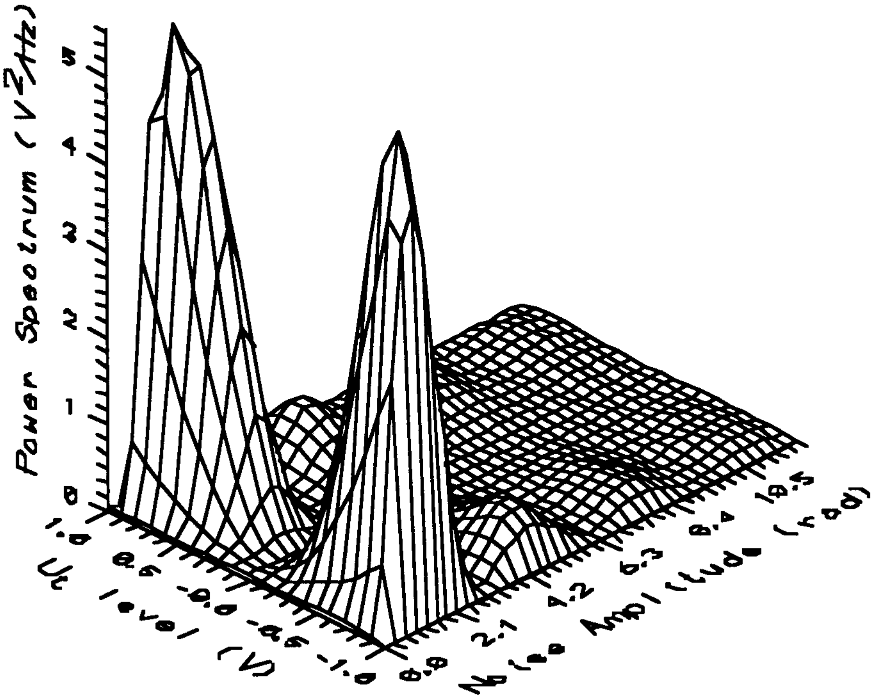

Figure 5: Power Spectrum at the signal frequency with various threshold levels and phase noise

4 Unsolved problems/questions

a. Theory. This is the really unsolved problem here. There is no theoretical explanation of these new results at the moment.

Some open questions:

b. The same effect with different noises and different signals.

c. Importance of this effect at practical applications. LCD systems are often used in the information technology and the occurrence of phase noise is rather general there, too. Whenever the threshold level of the LCD is not zero, stochastic resonance can probably be used to optimise the signal transfer.

Acknowledgements

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