

## A Novel Resonance in *n*-GaAs Diodes

Yuo-Hsien SHIAU\* and Zoltán NÉDA<sup>1</sup>

National Center for Theoretical Sciences (Physics Division), 101, Section 2 Kuang Fu Road, Hsinchu, Taiwan 300, ROC

<sup>1</sup>Department of Theoretical Physics, Babes-Bolyai University, strada Kogălniceanu nr.1, RO-3400, Cluj-Napoca, Romania

(Received May 28, 2001; accepted for publication July 25, 2001)

We consider two external laser beams interfering inside a *n*-GaAs with a nonlinear feedback. In a given parameter region coexistence of the two unstable periodic domain trains (UPDTs) is found in this system. The UPDT is associated with the formation of intermittent oscillating current in the circuit. By external periodic driving, a novel resonance is observed in both inside one UPDT and the switching between the two UPDTs. The signal-to-noise ratio (SNR) as a function of driving amplitude *A* is numerically investigated and the nonmonotonic behavior of the SNR(*A*) spectrum is observed.

KEYWORDS: semiconductors, resonance, communication, Gunn diode, simulation

The light-triggered Gunn-domain structure has recently made a significant impact in the field of photorefractive semiconductors.<sup>1)</sup> The concept is based on application of a strong electric field in the region of negative differential conductivity of multivalley semiconductors and consideration of the spatial modulation due to two external laser beams interfering inside semiconductors. In this case multiple high-field domains and the spatially modulated refractive index, i.e., optical grating, can be formed simultaneously in the material. If these two laser beams are under a feedback control, a new instability which changes the distance between adjacent domains will appear.<sup>2)</sup> In simple words, when the distance between adjacent domains is fixed, there is considered to be a stable periodic domain train (SPDT) in the material, which corresponds to oscillating current with a fixed frequency around that of microwaves in the circuit and the wavelength of the microwaves is constant over time. If the distance between adjacent domains is a function of time, then an unstable periodic domain train (UPDT) and the intermittent oscillating current will appear. Therefore, the variation of wavelength of the microwaves can be periodic or chaotic, which corresponds to periodic UPDT or chaotic UPDT, respectively. Besides, it is also interesting to observe that different SPDTs and/or UPDTs can coexist under the same physical conditions.

Resonant phenomenon is an interesting and important topic in dynamical systems. The issue of finding nonlinear resonances is closely related to the optimal control of dynamical systems.<sup>3)</sup> In general, there are two different kinds of resonance in nature. One is classical resonance (CR)<sup>4)</sup> and the other is stochastic resonance (SR).<sup>5)</sup> The fundamental differences between CR and SR are as follows: i) When a linear damped oscillator is subjected to an external periodic force, the oscillating amplitude or the absorption rate of energy as a function of external driving frequency *f* will display a resonant peak at  $f = f_0$ , where  $f_0$  is the natural frequency of the linear oscillator. This type of resonance characterized by external driving frequency is called CR. ii) SR is a phenomenon for bistable systems driven by a stochastic force-field and periodic modulation. On computing the signal-to-noise ratio (SNR) for the system's response to periodic driving as a function of the stochastic field intensity  $D_{st}$ , a characteristic maximum is observed. However, it is well known that there are very strong relationships between deterministic chaos and external periodic modulation. For example, it is possible to find driven chaos in dynamical systems.<sup>6)</sup> In other words, periodic

modulation can induce randomness. Therefore, the general formulation for SR with the coexistence of two chaotic states shall be expressed as<sup>7)</sup>

$$\text{SNR} \propto \frac{A^2}{(D_{st} + D_0)^2} \exp\left[\frac{-\delta V}{(D_{st} + D_0)}\right], \quad (1)$$

where *A* is the modulation amplitude,  $D_0$  is the deterministic noise intensity, and  $\delta V$  is the potential barrier. Thus if there is no external stochastic force field in the dynamical system, i.e.,  $D_{st} = 0$ , the only free parameter in eq. (1) is *A*, and  $D_0$  is dependent on the modulation amplitude, i.e.,  $D_0(A)$ . Therefore we can investigate SNR as a function of *A*. Please note that the physical meaning of eq. (1) is that a very weak periodic modulation is considered which cannot change the noise level. Then, SNR shall be proportional to  $A^2$ , which is treated as a nonresonant case. However, if *A* is sufficiently large, the relationship between  $D_0$  and *A* has no simple form. It is because the chaotic dynamics can possibly be suppressed or enhanced by the periodic driving. Therefore, the nonmonotonic behavior of SNR(*A*) spectrum is expected, i.e., resonant case.

In Fig. 1, the electric circuit feedback generates a nonlinear current function which will tune the incidence angle of the laser beams. The current equation for the electric circuit feedback is

$$I_n[\lambda(t)] = I_p[\lambda(t - T)] - I_l[d\lambda(t)/dt], \quad (2)$$

where  $I_p$  is the induced current from the microwave power, detected by the detector (see Fig. 1),  $I_l$  is the current loss in

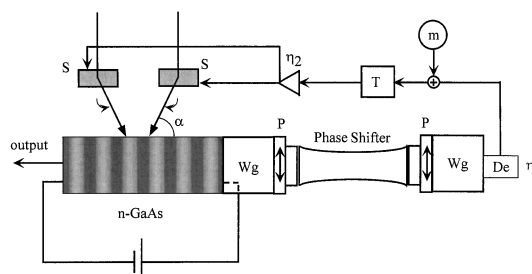


Fig. 1. Proposed experimental setup: the feedback loop consists of a pair of waveguides *Wg*, a phase shifter whose fast and slow axes are at 45° to a pair of crossed polarizers *P*, a detector *De* with a time response of approximately 10 ns, modulation current *m*, a delay line with retardation time *T* of approximately value *ms*, and a pair of acousto-optic scanners *S* to tune the incidence angle  $\alpha$  of laser beams. The purpose of *De* is to detect microwave power and to convert the nonlinear power into electric current. The phase shifter is used to generate the nonlinear power function.

\*Corresponding author. E-mail: yhshiau@phys.cts.nthu.edu.tw

the circuit, and  $I_n$  is the net current for driving a pair of acousto-optic scanners which will tune the incidence angle of the laser beams. Usually,  $I_1$  is much smaller than  $I_n$  and  $I_p$ , thus  $I_1$  can be neglected in eq. (2). In ref. 2, we show that  $I_n$  and  $I_p$  are proportional to  $\lambda(t)$  and  $\lambda^2(t - T) \sin^2[\pi D/\lambda(t - T)]$ , respectively, where  $T$  is the delay time and  $D$  is the length of the phase shifter. For the convenience of analysis, we treat eq. (2) as a difference equation, i.e.,  $\lambda_n \equiv \lambda(t + nT)$  and  $0 < t < T$ , and reduce it to be dimensionless, i.e.,  $\lambda_n/\lambda_0 \rightarrow \lambda_n$ ,

$$\lambda_n = 1 + \beta \lambda_{n-1}^2 \sin^2 \left( \frac{\pi D}{\lambda_0} \frac{1}{\lambda_{n-1}} \right). \quad (3)$$

In the above discrete dynamics formula,  $\beta$  is considered as the effective microwave power and  $\lambda_0$  is the initial wavelength, determined by the initial incidence angle of the beams with no feedback control.

The discrete dynamical system exhibits very rich behaviour as a function of the two relevant parameters  $\beta$  and  $D/\lambda_0$ . Choosing their value properly, one, two or several SPDTs, periodic UPDTs, chaotic UPDTs or a coexistence of these can be found.<sup>2)</sup> For our present study, we consider the case  $D/\lambda_0 = 2.3$  and  $\beta = 0.4$  to determine the coexistence of three PDTs. One is a SPDT and the other two are chaotic UPDTs via a period-doubling route. In order to study the resonant effect, we add now a slow modulation current  $I_m$  to the dynamical system. The new difference equation governing the dynamics in our systems is

$$\lambda_n = 1 + \beta \lambda_{n-1}^2 \sin^2 \left( \frac{\pi D}{\lambda_0} \frac{1}{\lambda_{n-1}} \right) + A \cos(2\pi f n). \quad (4)$$

The numerically computed SNR is defined as

$$\begin{aligned} \text{SNR} &= \frac{\lim_{\Delta \bar{f} \rightarrow 0} \int_{f-\Delta \bar{f}}^{f+\Delta \bar{f}} [S(\bar{f}) - S_0(\bar{f})] d\bar{f}}{S_0(f)} \\ &\propto \frac{1}{N} \frac{S(f) - S_0(f)}{S_0(f)} \equiv \frac{R}{N}, \end{aligned} \quad (5)$$

where

$$R = \frac{S(f) - S_0(f)}{S_0(f)} \quad (6)$$

$S_0(\bar{f})$  is the one-sided power spectrum of the noisy background,  $S_0(f)$  is the power of the deterministic noise at the driving frequency, and  $S(\bar{f})$  is the power of the total response.  $N = 2^p$  ( $p = 1, 2, \dots$ ) is the length of the input  $\lambda_n$  series we investigate.  $R$  is the height ratio of net signal power and

noisy power at the driving frequency. In Fig. 2, considering four different driving frequencies, we plot  $R/N$  as a function of the driving amplitude  $A$ . Two different regimes are observed. For  $A < 0.37$ , the system is trapped inside one chaotic UPDT (i.e., inside its own basin), while for  $A \geq 0.37$  the system can jump between the two chaotic UPDTs. In the first regime for  $0 < A < 0.17$ , the system is trapped inside the first chaotic UPDT (with lower values of  $\lambda$ ) and for  $0.37 > A \geq 0.17$  in the second chaotic UPDT. (All state limits are given only with a precision of two decimal digits.) At the lower driving frequencies (i.e.,  $f = 0.001$  and  $0.0001$ ), we can observe that  $R/N$  exhibits the expected scaling from eq. (1), i.e.,  $R/N \propto A^2$ , when the system is flipping between the two chaotic UPDTs. It implies that  $D_0$  is not sensitive to the variation of the driving amplitude at the lower driving frequencies. In other words,  $D_0$  in this case can be considered as a fixed value determined from the original chaotic dynamics. However, when we increase  $f$  to 0.1, both a resonant peak as a function of driving amplitude and  $R/N \propto A^2$  scaling after the appearance of resonant peak occur. Therefore, in the resonant regime, the intermittent chaos-chaos transition can be optimally synchronized with the external modulation, which is much better than the synchronization via  $R/N \propto A^2$  scaling. When  $f$  is fixed at 0.01, a clear resonant peak is observed while the dynamical system is trapped inside one chaotic UPDT. It is interesting to find that  $R/N$  fast decays to very small values after the resonant peak. This means that the periodic signal can be optimally encoded in the chaotic output. This result can be applied to private communication.

The above results show that the nonmonotonic  $\text{SNR}(A)$  spectrum can be obtained in our proposed microwave system. Not only in the flipping between the two chaotic UPDTs but also inside one chaotic UPDT, resonant peaks are observed. To our knowledge, this kind of resonance is unique. We think that  $\text{SNR} \propto A^2$  scaling is a very intuitive concept when the external modulation is applied to the chaotic system. However, this nonresonant concept is only valid at the lower driving frequencies in our system, which is demonstrated by our numerical study. Our results are different from those for CR and SR. For CR, there is a resonant peak in the  $S(f)$  spectrum but no characteristic maximum is observed in the  $S(A)$  spectrum, which always displays monotonic  $S \propto A^2$  scaling. For SR, the resonant peak is observed in the  $\text{SNR}(D_{st})$  spectrum not in the  $\text{SNR}(A)$  spectrum.

This work was supported in part by the National Science Council of the Republic of China under contract nos. NSC 88-2112-M-002-002.

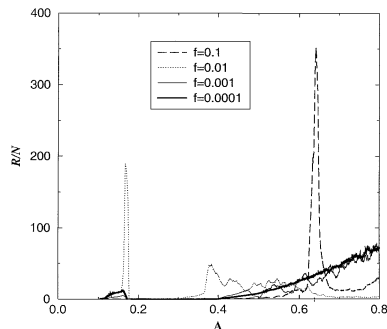


Fig. 2.  $R/N$  as a function of driving amplitude considering four different driving frequencies.

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