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Critical behavior of a spring-block model for magnetization

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The critical behavior of a one-dimensional spring-block model aimed to describe magnetization phenomena is studied by Monte-Carlo type computer simulations. The introduced model resembles the classical Burridge-Knopoff type models where the blocks represent the Bloch-walls that separate inversely oriented magnetic domains, and springs correspond to the magnetized regions. Disorder is introduced through randomly distributed pinning centers along the sample and the magnetization process is modeled through a relaxational dynamics. The shape of the hysteresis loops and the distribution of avalanche sizes are studied (jumps in magnetization) for different disorder values. As a function of the amount of disorder in the system, in agreement with previously introduced magnetization models, the subcritical, critical and supercritical regions are identified. The results indicate that for a critical amount of disorder the introduced model exhibits critical behavior characterized by power-law distribution of the avalanche sizes. An estimation of the critical exponent is given.

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1. Introduction

The study of disordered systems is an important research area in statistical and condensed-matter physics. Disorder-induced first-order phase transitions [1-8] represent a very fashionable problem in this field. Whenever a phase transition takes place in a physical system, the first or higher order derivatives of the free energy functional becomes non-analytical. Accordingly, some characteristic thermodynamic properties diverge or have singularities. In the vicinity of a phase transition the system exhibits critical behavior characterized by power-law type divergence or convergence to zero of some thermodynamic quantities. The exponents characterizing these power-laws (critical exponents) are universal in some sense. They do not depend on the microscopic details of the system, only on its dimensionality and symmetry properties. Universality is due to the fact that near criticality the correlation length in the system becomes very large, even infinite, thus making it possible to average out many microscopic degrees of freedom.

For some types of first-order phase transitions thermal fluctuations are secondarily small. This is the reason why they are called fluctuationless phase transitions. Such tansitions are for example the athermal solid-solid diffusionless martensitic transitions[1] [9], and the field-induced first-order phase transitions in ferromagnetic systems where the external driving field ($\boldsymbol{H}$) plays the role of the driving force in the transition. A typical model of such type of first-order phase transition is the Ising system in external magnetic field $\boldsymbol{H}$ at $\boldsymbol{T}<\boldsymbol{T_c}$. When $\boldsymbol{H}$ changes sign, the magnetization reverses abruptly. The difference between conventional fist-order phase transitions (like for example liquid-gas transition) and solid-solid transition (when the material changes its crystalline or magnetic form) is that in the latter case the sharp transition is absent. Hysteresis occurs instead. Hysteresis occurs even if the system is driven extremely slowly. This fact shows that the occurrence of hysteresis is not of kinetic nature, but it is due to the quenched disorder present in the system.

In general the presence of the disorder changes the free-energy landscape of the system in a way that there will be energy barriers separating many local minima (metastable states). In magnetic systems the movement from one metastable state to the next one is a collective process, involving a number of magnetic domains. This is called avalanche. Avalanches lead for example to the magnetic Barkhausen noise (BN). When the distribution of the avalanche sizes obeys power-law, this indicates the existence of criticality in the system. Such transition is related to the change of the properties of the hysteresis loop and avalanche size distribution as a function of the disorder. This is the reason why these types of transitions are called disorder-induced. In the vicinity of the critical point the main differences relative to the classical phase transition are: (i) the system has a history-dependent metastable evolution (is not in equilibrium), and (ii) the process is deterministic at $\boldsymbol{T = 0}$, thus no thermal fluctuations are present.

Several Ising-type models that are able to account for disorder-induced criticality in magnetic systems were already introduced and studied [1,5,10]. Ferromagnetic coupling, interaction with the external driving magnetic field and effect of some kind of disorder governs the

[1] Martensitic transitions occur in a wide range of conventional materials including high carbon steels. It is a first-order diffusionless phase transition when the triangular crystalline structure shears to square symmetry through local elastic distortions.

dynamics and time-evolution of these systems. A general Hamiltonian could be written in the following shape:

$$H_{gen} = \sum_{\langle ij \rangle} J_{ij} s_i s_j + H \sum_i s_i + disorder \qquad (1)$$

Depending on the manner in which disorder is taken into account in the Hamiltonian, three different models were intensively studied. The *random-field Ising model* (RFIM) introduced and studied by Sethna et al. [1-3] takes into account the effect of disorder in the Ising-system through a local random field acting on each site of the lattice. The *random-bond Ising model* (RBIM) was introduced by E. Vives and A. Planes in Ref. [5]. As the name of the model indicates, randomness is introduced in the system through random coupling constants ($\boldsymbol{J_{ij}}$ in the Hamiltonian (1)). Vives and Planes studied also another model aimed to describe magnetization phenomena, namely the *random anisotropy Ising model* (RAIM) [10] In this case a random anisotropy is responsible for the disorder.

Simulations and analytical calculations were performed on these models. Regarding the occurrence of critical behavior the results are in qualitative agreement with each other. For a certain well-defined value of the disorder the results indicate the presence of a conventional critical point in the system. Below this critical amount of disorder the magnetization reversal from negative to positive saturation is abrupt. Above the critical value of disorder the magnetization reversal takes place continuously without a giant avalanche which spans the whole system. There is thus a well-defined value of the quenched disorder in the system for which the transition between sharp and smooth magnetization reversal takes place. At this point a first-order fluctuationless phase-transition occurs. The tunable parameter is the amount of disorder and the most relevant signature of the critical state is the occurrence of power-law distributions for some characteristic quantities like avalanche-size distribution, signal energy-, duration- or area.

Many other conceptually different models were also elaborated in order to capture the occurrence of criticality and power-laws in magnetization phenomena. Some of them considers the motion of the domain walls which separate magnetic domains with different orientation (see for ex. Refs. [11-17]). These models consider different types of interactions, giving certain contributions to the free-energy functional of the studied system: exchange energy, magnetostatic energy, magnetocrystalline anisotropy, magnetoelastic energies and disorder. For a complete review see Ref. [18].

After theoretically predicting the possibilities of disorder-induced phase transitions, one might put the legitimate question whether this is a real phase transition observable in real materials. Recently Berger et al. [6] provided experimental evidence for disorder-driven phase transitions. Berger et al. reports measurements performed on an exchange-coupled *Co/CoO*-bilayer structure. The *Co* is ferromagnetic with Curie temperature T_C*=1388 K*, the *CoO* is antiferromagnetic with Néel temperature T_N*=291K*. The experiment was performed between *T = 80-300 K*. This bilayer system has the great advantage that the degree of the disorder can be reversibly varied. The interface roughness together with the interface exchange coupling between the two layers results in an effective disorder in the system. Varying the temperature, experimentalists could influence the anti-ferromagnetic ordering of the *CoO* layer. Interface imperfections cause spin frustration near the Néel temperature and induce magnetic disorder. Thus, they could control the effective disorder in the bilayer system simply by tuning the temperature. Near *T = 190 K* a dramatic change takes place: the magnetization of the sample varies continuously, while for higher temperatures (from *T = 200 K*) the magnetization of the whole sample reverses abruptly. This transition cannot be a consequence of thermal effects because at this temperature both layers are below their critical temperature. By the other hand Ref. [6] reports also a reference measurement with a single *Co* layer, which instead showed discontinuous magnetization reversal even at low temperatures. This measurement proved that the disoder induced by the interface-coupling is responsible for the critical behavior. Scaling analysis proved the existence of scaling behavior in the vicinity of the critical point. Experiments established thus the existence of a conventional critical point associated with the amount of magnetic disorder.

In this paper we investigate the possibility and necessary conditions for the occurrence of critical behavior in a simple one-dimensional spring-block model for magnetization. This model was recently introduced by us, and successfully used to describe the statistics of the Barkhausen noise [19].

2. Theoretical model. The spring-block model

The model is essentially a one-dimensional (1D) spring-block system, similar to the 1D Burridge-Knopoff models [20,21] applied in the study of earthquakes. It is aimed to reproduce the accepted microscopic picture of domain wall dynamics for 180 degree Bloch-walls which separate inversely oriented (+ | - | + | - | + …) magnetic domains (Fig. 1).

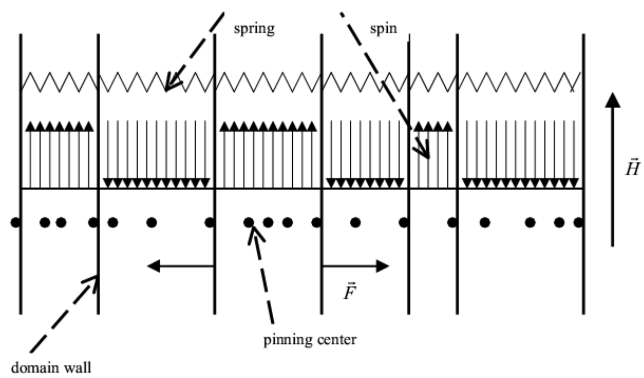

Fig. 1. Sketch of the mechanical spring-block model.

We assume that the domain walls are pinned by defects and impurities, and cannot move unless the resultant force acting on them is bigger than the strength of the F_p pinning force. When the resulting force is greater than the pinning force, the wall simply jumps in the resulting force direction on the next pinning center. Apart of this pinning force there are two other types of forces acting on each domain wall. To understand these forces let us consider the *i*-th wall (which separates the (*i-1*)-th and *i*-th domain) free to move and all other walls fixed. One of the forces acting on the domain wall, F_H, results from the magnetic energy of the domains *i* and (*i-1*) in an external magnetic field H and has the form:

$$F_H = (-1)^i \beta \cdot H, \quad (2)$$

with β constant. In our model for the sake of simplicity we define the units such that $\beta=1$. For positive values of the external magnetic field this force encourages the increase of the domains oriented in the + direction, and for negative values of the external magnetic field this force tends to increase the size of the domains oriented in the - direction.

A second type of force, F_m, acting on both sides of the domain walls, is due to the magnetic self-energy of each domain. This force tends to minimize the length of each domain. It can be shown that F_m is proportional with the length of the considered domain x_i.

$$F_m = -f_m x_i. \quad (3)$$

The constant f_m is an important coupling parameter in this model and acts as the elastic constant of a mechanical spring.

The system of the F_p, F_m and F_H forces can be now easily mapped on a one-dimensional spring-block Burridge-Knopoff type model [20].

The main constituents in this mechanical model are randomly distributed pinning centers, rigid walls sitting on pinning centers (describing Bloch-walls) separating + and - oriented domains and springs between the walls (describing the F_m forces). The strength of the pinning centers (pinning forces), F_p, are randomly distributed following a normal distribution. This force is modeled as being similar to a static frictional force. Walls can be only on pinning centers and two walls are not allowed to occupy the same pinning center. This constraint implies that the number of magnetic domains and domain walls are kept constant and are thus a-priori fixed. Domains cannot totally disappear and new domains cannot appear during magnetization phenomena. The elastic springs are ideal with zero equilibrium length and with the tension linearly proportional with their length. The tension in the elastic springs will reproduce the F_m forces. Beside the pinning forces and the tensions in the springs there is an extra force acting on each wall. The strength of this force is proportional with the exterior magnetic field's intensity, it is the same for all walls but its direction is inverse for +|- and -|+ walls. This force will reproduce the F_H forces. The main differences relative to the classical Burridge-Knopoff type models [20,21] is that in our case the driving force acting on the blocks has different orientations for the first-nearest neighbors, and there is absent a second layer of springs which connects the sliding blocks to the driving force.

The dynamics of this model is aimed to reproduce real magnetization phenomena. First N_p pinning centers are randomly distributed on a fixed length (L) interval, and their strengths are assigned. Than a fixed N_w number of walls are randomly spread over the pinning centers ($N_w \ll N_p$) and connected by ideal springs. Neighboring domains are assigned opposite magnetic orientation. The system constructed this way will be driven through several whole magnetization-demagnetization cycles (hysteresis loops) using relaxational dynamics. This dynamics has two main laws: (i) if $|FH+Fm| > F_p$ is true in the case of wall labeled *i*, it will jump to the next pinning center in the direction of the resultant force, except (ii) the next pinning center is occupied by another domain-wall. In this case wall *i* will stay in its original place. The system has reached equilibrium if no wall can move anymore. We assume that the time needed for the system to achieve equilibrium is zero. It is important to note that one event (jump) can trigger many other events leading to avalanche-like processes. The order in which the position of the walls is updated is random. The value of the F_H external force is increased step-by-step (corresponding to an increasing H magnetic field intensity), and for each new F_H value an equilibrium position of the system is searched. In each equilibrium configuration we calculate the total magnetization of the system as:

$$M = \sum_i l_i \cdot s_i, \quad (4)$$

where l_i is the length of domain *i*, and s_i is it's orientation: +1 for positive orientation, and -1 for negative orientation.

During the simulation we are monitoring the variation of the magnetization focusing on the shape of the hysteresis loop and jump-size distribution. The hysteresis loop is the history-dependent relation between the magnetization M and the external magnetic field H when the value of H is increased and decreased successively. The jump size distribution ($g(s)$) is the distribution function for the obtained values of abrupt jumps in M throughout many hysteresis loops.

The parameters of the model are: N_p – the number of pinning centers; N_w – the number of Bloch-walls (usually $N_w \ll N_p$); the geometrical size of the sample L which determines the density of the pinning centers; f_m – the coupling constant between the neighboring domain walls (corresponds to the elastic constant in the case of coupled springs); the dH – driving rate of the external magnetic field (change in H for one simulation step) and the standard deviation σ of the pinning forces.

3. Simulation results

In the present study we focus on the analysis of the hysteresis loops and the corresponding jump size distribution functions while the amount of disorder is varied in the system. Our goal is to investigate the possibility of obtaining a critical behavior in this model and thus the presence of a disorder-induced phase transition.

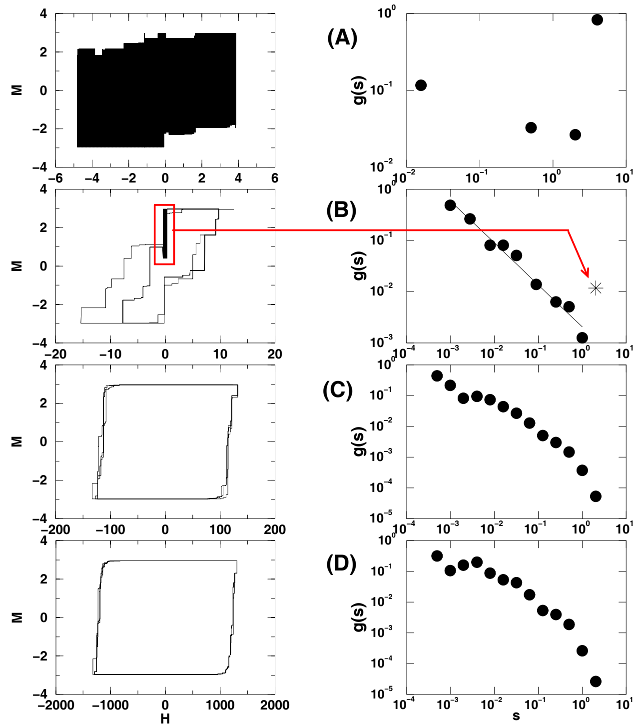

Fig. 2. Hysteresis loops (left column) and corresponding jump-size distributions (right column) for different amount of disorder in the system. Figures (A) are for σ =0.1, figures (B) are for σ =1, figures (C) are for σ =10 and figures (D) stand for σ =100. The power-law fit for the jump-size distribution on figure (B) indicates a scaling exponent -0.9. Other parameters of the simulations are: Np/Nw=100, Np = 1000, L = 3, fm = 10 and dH=0.001.

As it is known from previous studies on RFIM, RBIM and RAIM [1,5,10] the amount of disorder in the considered model has a crucial role on the statistical properties of the obtained magnetization noise. For low disorder and very strong disorder values the jump-size distribution does not exhibit a scaling property. Usually there is a critical amount of disorder for which the jump-size distribution has a power-law decay. In our model the amount of disorder can be controlled in two ways, either by varying the density of pinning centers (**N_p/L**) or by changing the standard deviation σ of the strength of their distribution function. In our study we followed the second way.

For different disorder levels the characteristic hysteresis loops and the corresponding jump-size distributions are plotted on Fig. 2. Since the jump size distribution histogram in our simulation corresponds to the avalanche size distribution from experiments, it is the most relevant distribution characterizing the statistics of this disorder induced phenomenon and in particular the Barkhausen noise.

For high disorder level (Fig. 2 (B), (C) and (D), σ ≥ 10) the shape of the obtained hysteresis curves satisfies our expectations and fulfills all the requirements for real magnetization phenomena. On these curves one can detect many discrete jumps with different sizes, thus the model exhibits BN. In addition, when the sample is driven consecutively through many hysteresis cycles the magnetization curves do not follow exactly the same path, although the parameters of the simulation were unchanged. The qualitative shape of the hysteresis curve is quite stable for a wide range of the free parameters. In contrast, for low disorder in the system (Fig. 2 (A), σ < 1) the hysteresis loops are damaged, and we have many back and forward jumps in the magnetization, since the equilibrium position of the system is quite hardly reached (this is the explanation for the filled and damaged hysteresis cycle for σ = 0.1). For the considered fixed **N_p/N_w = 100** value the jump-size distribution the simulated curves show that the critical behavior is reached for σ = 1. For this disorder value the jump-size distribution function follows an almost perfect power-law with exponent around ***-0.9***. On the hysteresis loop on this figure (Fig. 2B) one can still observe that sometimes there is a small regime where the system's magnetization jumps back and forward (the region in the drawn box). The last point drawn with a star on the corresponding jump-size distribution function is due to this unstable part of the hysteresis loop and seemingly is out of the scaling.

4. Conclusions

Comparison of the obtained simulation results with previous theoretical and experimental ones yields thus the following main conclusions:

1. In agreement with the previously used random-field, random-bond and and random anisotropy Ising models [1,5,10], as a function of the amount of disorder in the system the Burridge-Knopoff type spring-block model introduced by us suggests also the existence of three different regimes: subcritical, critical and supercritical.
2. The simulated hysteresis loops in the critical and supercritical regimes are qualitatively in good agreement with the ones obtained in experiments. One can observe many Barkhausen jumps with various sizes, just as it is expected from the experimental results. We can also learn from the graphs that the hysteresis loops do not follow the same path when the system goes through several cycles.
3. The shape of the simulated jump size distribution function predicts a nice scaling in the critical regime, and a non-power-law nature in the supercritical regime. Our results suggested in the critical regime a power-law fit with an exponent around ***-0.9***. Experimentally this quantity is usually not investigated, since it is difficult to detect those very small changes in the magnetization.

From all these scaling properties we conclude that the power-law tendencies suggested in our simulations can explain at least qualitatively the measured statistics of the Barkhausen noise. The simple one-dimensional mechanical model presented here is suitable thus to qualitatively reproduce real magnetization phenomena with special attention to magnetic Barkhausen noise. The model captures the main elements of the microscopic dynamics for the phenomenon and in spite of its gross simplification, it contains all the necessary ingredients to account for the critical behavior of magnetic systems governed by quenched disorders. The model is also suitable for pedagogical purposes and can be easily implemented on computer.

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