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Unexpected synchronization

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Abstract. An ensemble of oscillators capable of a highly nontrivial synchronization is investigated. The oscillators are stochastic units which can emit a signal and detect signals emitted by others. They have two operational modes characterized by two different oscillation periods. Switching between the modes is governed by a simple rule: the average output intensity is kept around a prescribed threshold. This simple dynamical rule realizes the coupling of the units and leads to a complex collective behaviour. Computer simulations proved that for a given interval of the threshold parameter, partial synchronization of the units occurs. The appearance and disappearance of this synchronization as a function of the threshold parameter indicates a phase-transition type behaviour.

1. Introduction

Spontaneous synchronization occurs in diverse systems in nature. Examples include biological systems such as fireflies flashing in unison or crickets chirping together, rhythmic applause, mechanically coupled metronomes, the menstrual cycles of women living together, pacemaker cells in the heart, etc. [1]

Several mathematical models have been put forward to explain how a large group of coupled oscillators with different frequencies can synchronize spontaneously, without an external driving force. Most of these models fall into two broad categories: those describing phase-coupled oscillators, and those which use pulse-coupling between the units. In the present work we shall take a look at a totally different type of synchronization mechanism: one in which the interaction between the oscillating units is not chosen explicitly to induce synchronization. Instead of being the result of an evident phase-difference minimizing force, synchronization arises as a side effect of a simple optimization rule. Similar models are described in [2, 3, 4].

The classical phase-coupled model of synchronization was introduced by Kuramoto and Nishikawa [5]. In their model, each oscillator has an associated phase between 0 and 2π . The oscillators evolve according to a set of coupled first order differential equations, with a coupling that minimizes the phase difference between them. The form of the equations was chosen to allow for an analytic solution. In the thermodynamic limit, this model shows a second-order phase transition as a function of the coupling strength. The critical coupling depends on the variance of the oscillators' frequencies.

Pulse-coupling is used in integrate-and-fire type models [6]. Each oscillator has a state variable, which increases monotonically until it reaches a given value. At this point the oscillator emits

a pulse (it *fires*) and resets its state. The state variable of all those other oscillators that can detect the pulse increases instantly by a fixed value. Under the right conditions, a single pulse can trigger an avalanche of pulses in the system, thus causing a high proportion of the units to fire at the same time.

2. The Model

The model consists of an assembly of N similar oscillators, each of which can either be active at a particular instant in time, outputting a signal of strength $1/N$, or inactive, outputting no signal at all. Thus the total output signal of the assembly can vary between 0 and 1. It is easy to picture these oscillators as flashing units, so the active oscillators will be referred to as *lit*, while the inactive ones will be called *unlit*. Correspondingly, the total output level of the system can be thought of as the total light intensity.

The units are two-mode stochastic oscillators, i.e. they can oscillate with a longer or a shorter period, and these periods are random variables. At the end of a period, oscillators stay lit for an amount of time.

Each oscillator cycles between three states, which will be denoted by A , B and C (see Figure 1a) [2, 3, 4]. The length of time a unit stays in each of the three states will be denoted by τ_A , τ_B and τ_C .

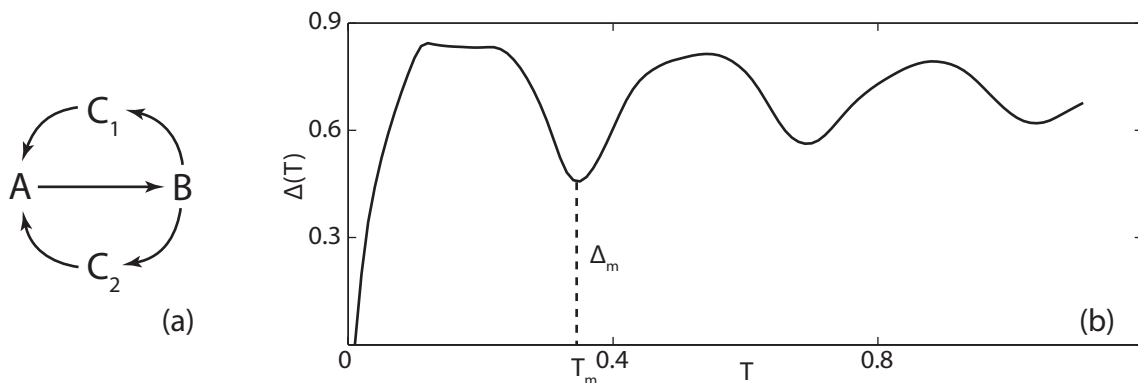


Figure 1. (a) Dynamics of the oscillators. Each oscillator cycles between three states, A , B and C . The duration of phase C can be long or short (C_1 and C_2). (b) Shape of the $\Delta(T)$ function for parameters $f^* = 0.2$, $\tau^* = 0.05$ and $N = 2000$.

State A is the stochastic phase of the oscillators. Its length, τ_A , is a random variable following an exponential distribution:

$$P(\tau_A) = \frac{1}{\tau^*} e^{-\tau_A/\tau^*} \quad (1)$$

The average duration of state A is $\langle \tau_A \rangle = \tau^*$.

Phase B has a fixed length τ_B . This ensures that the oscillator stays unlit for at least a time of τ_B .

State C is the lit state of the units, and can have two durations, a longer one, τ_{C1} , and a shorter one, τ_{C2} . These correspond to oscillating modes 1 and 2, respectively. When entering state C , the oscillators decide how long to stay in that state based on the total output of the system. If the output f is less than a prescribed value f^* , the oscillator will choose mode 1. If $f > f^*$, it will choose mode 2.

By choosing mode 1 when $f < f^*$ and choosing mode 2 when $f > f^*$, the units try to keep the output close to f^* . As a side effect of this optimization, for certain values of the model

parameters the oscillators will flash synchronously, and the total output of the system becomes approximately periodic in time.

This model is a variant of the models presented in [2, 3, 4]. In the earlier models, instead of phase C, it was the length of phase B that was different between the two modes. Note that unlike in the earlier versions, in the present model the length of a full oscillation period is lengthened rather than shortened by a low total output.

To back up the claim that the units are capable of flashing in unison, we need an objective way to detect synchronization in the system. For this, a $\Delta(T)$ function that characterizes the periodicity level of the total $f(t)$ output was chosen [2, 3]:

$$\Delta(T) = \frac{1}{2M} \lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x |f(t) - f(T)| dt \quad (2)$$

where

$$M = \lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x |f(t) - \langle f(t) \rangle| dt \quad \text{and} \quad \langle f(t) \rangle = \lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x f(t) dt \quad (3)$$

The shape of this function is sketched on Figure 1b. The more periodic the output signal is (assuming a period T), the smaller the value of $\Delta(T)$ will be. Thus the period of the approximately periodic function $f(t)$ can be considered to be T_m , where $\Delta_m = \Delta(T_m)$ is the deepest minimum of $\Delta(T)$ (excluding the obvious minimum at $T = 0$). It is easy to see that for a perfectly periodic function $\Delta_m = 0$, and that Δ_m does not depend on the amplitude of the signal. We can define the *periodicity level* of the output as $p = 1/\Delta_m$.

The quantity p/p_1 was chosen as the order parameter characterizing the synchronization level of the assembly of oscillators. Here, p_1 is the periodicity level of a single unit oscillating in the long mode (τ_{C1}), hence p/p_1 is the increase in the periodicity level of the output due to the coupling between the oscillators.

3. Results

The model was implemented on a computer, and studied numerically using simulations. The level of synchronization as a function of τ^* and f^* was studied. The other parameters of the model were fixed at the following values: $\tau_B = 0.20$, $\tau_{C1} = 0.15$ and $\tau_{C2} = 0.10$. (Other choices lead to qualitatively similar results.)

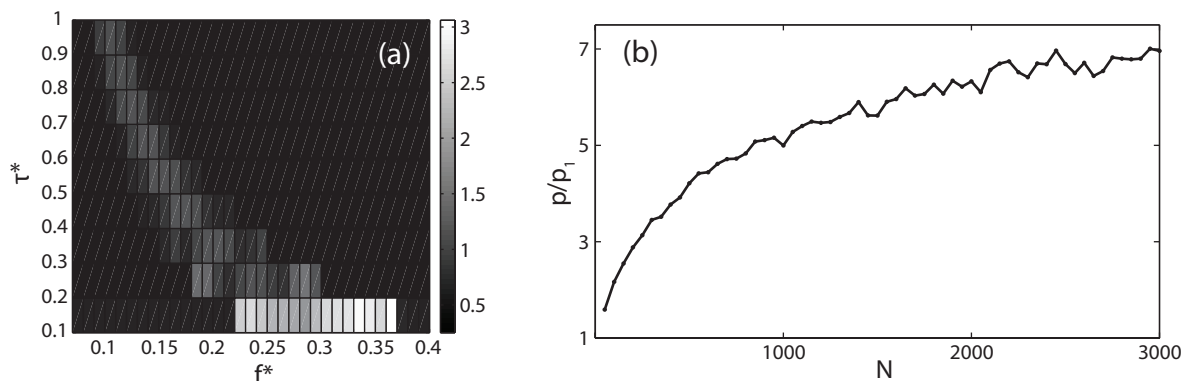


Figure 2. (a) Synchronization level as a function of τ^* and f^* . Lighter shades of grey indicate a higher p/p_1 value. Synchronization occurs only in a certain region of the f^* - τ^* space. (b) Synchronization level versus the number of oscillators.

Similarly to earlier models, where the modes differed in the value of τ_B , the present model exhibits synchronization in an island-like region in τ^* - f^* space (Figure 2a). It was also found

that in this region, the synchronization level, as measured by p/p_1 , increases monotonically with the number of units in the system (see Figure 2b).

For a fixed τ^* , synchronization appears and disappears abruptly as f^* is varied (see Figure 3a). The transition gets sharper as N is increased, suggesting a phase-transition like phenomenon.

The distribution of the followed oscillation modes is plotted on Figure 3b for a few values of f^* . As expected, for those values of f^* where synchronization occurs, both modes are present. For low or high f^* , solely one mode occurs, barring any possibility of non-random shift in the length of oscillation cycles and thus synchronization.

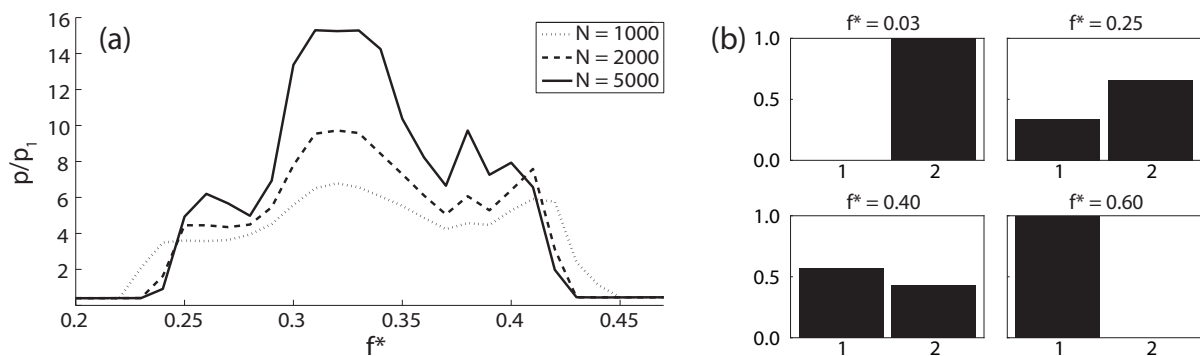


Figure 3. (a) Synchronization level as a function of f^* . τ^* was fixed at 0.05, and the number of units, N , is indicated in the legend. (b) Distribution of the oscillation modes for various values of f^* . Results obtained for $\tau^* = 0.05$ and $N = 2000$.

4. Conclusions

An assembly of two-mode stochastic oscillators were investigated by computer simulations. At the end of its oscillation period, each unit emits a signal of either long or short duration (corresponding to the two modes), depending on the total output intensity in the system. When the light intensity is less than a threshold value, f^* , the system emits a long signal, otherwise a short one. This dynamics tries to keep the average output around f^* . As an unexpected side effect, for certain values of f^* synchronization occurs. The abrupt appearance and disappearance of this synchronization resembles a phase transition. Synchronization is improved by increasing the number of units in the system.

The synchronization mechanism found in this study is interesting and puzzling because no phase-difference minimizing interactions are involved in the dynamics.

Acknowledgments

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