Collective behavior of electronic fireflies

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Abstract. A simple system composed of electronic oscillators capable of emitting and detecting light-pulses is studied. The oscillators are biologically inspired, there are designed for keeping a desired light intensity, W, in the system. From another perspective, the system behaves like modified integrate and fire type neurons that are pulse-coupled with inhibitory type interactions: the firing of one oscillator delays the firing of all the others. Experimental and computational studies reveal that although no direct driving force favoring synchronization is considered, for a given interval of W phase-locking appears. This weak synchronization is sometimes accompanied by complex dynamical patterns in the flashing sequence of the oscillators.

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1 Introduction

Synchronization of quasi-identical coupled oscillators is one of the oldest and most fascinating problems in physics [1–3]. Its history goes back to Huygens who first noticed the synchronization of pendulum clocks hanging on the same wall. Besides mechanical or electric oscillator systems, nature has also several amazing examples in this sense [4–6]. Synchronization in all these systems appears as a result of some specific coupling between the units. This coupling can be local or global, and can be realized through a phase-difference minimizing force [7–9] or through the pulses emitted and detected by the oscillators [10–12]. In most of these synchronizing systems there is a clear driving force favoring synchronization, and in such way the appearance of this collective behavior is somehow trivial. In the present work however, a nontrivial synchronization will be presented. This weak synchronization (phase-locking) appears as a co-product of a simple optimization rule.

One well-known phenomena which inspired us in this work is the collective behavior and synchronization of fireflies [13]. Although our aim here is not to model fireflies, the oscillators ("electronic fireflies") considered in our system are somehow similar to them: they are capable of emitting light-pulses and detecting the light-pulse of the others. In this sense our system is similar to an ensemble of fireflies although the coupling between the units is different. From another perspective, the oscillators behave like pulse-coupled "integrate and fire" type

neurons [10–12,14]. Contrary to the classical integrate and fire oscillators, in the considered system an inhibitory type global interaction is considered. This means that the firing of one oscillator delays (and not advances) the phase of all the others.

This system does not necessarily favor synchronization, it is rather designed to keep a desired W light intensity in the system. This light intensity is controlled by a firing threshold parameter G imposed globally on the oscillators. Surprisingly, as a co-product of this simple rule, for certain region of the firing threshold parameter phase-locking and complex patterns in the flashing sequence of the oscillators will appear

Integrate and fire oscillators with inhibitory type coupling were already studied by Ernst, Pawelzik and Geisel [15,16]. Their study revealed that such system are capable of complex collective behavior, exhibiting under quite general conditions a multistable phase clustering. Our oscillators are however different from the ones considered by the earlier studies, because the interaction results from an optimization rule, which then induces an inhibitory coupling. In this sense our system is much simpler, and the built in desire to optimize a global output intensity could be characteristic for several real systems as well. One candidate for such system could be neurons under some specific conditions. It is known that neurons do synchronize their firing [17] and it is also believed that inhibitory type coupling could be present in these systems [18].

The studied system will be described in more details in the following sections. The used electronic device will

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be briefly presented and the obtained non-trivial collective behavior will be studied. In order to get more confidence in the observed non-trivial results computer simulations are also performed.

2 The experimental setup

The constructed units are integrate and fire type oscillators [12] with a modified interaction rule. Their coupling and communication is through light, the units are capable of emitting and detecting light-pulses. The oscillators are practically realized by a relatively simple circuit, the main active elements being a photo-resistor and a Light Emitting Diode (LED). Each oscillator, i, has a characteristic voltage U_i, which depends on the resistance, R_i, of its photo-resistor. The global light intensity influences the value of R_i in the following sense: when the light intensity increases R_i decreases, leading to a decrease in U_i. In the system there is a global controllable parameter G, identical for all oscillators. By changing the parameter G, one can control the average light intensity output, W, of the whole system. If the voltage of the oscillator grows above this threshold ($U_i > G$) the oscillator will fire, this meaning its LED will flash. This flash occurs only if a minimal time period T_{min_i} has expired since the last firing. The oscillator has also a maximal period, meaning if no flash occurred in time T_{max_i}, then the oscillator will surely fire. In laymen terms firing is favored by darkness and the value of the controllable G parameter characterizes the "darkness level" at which firing should occur. Through this simple rule the G parameter controls the average light intensity output of the system.

The technical realization of the above dynamics is illustrated in Figure 2. After the system has fired the 22 μF capacitor is completely discharged ($U_c = 0$) by the negative pulse from the inverted output of the monostable. As soon as the light flash ended, the same capacitor will start charging from the current flow through the 270 KΩ resistor. The IC1B comparator will trigger another flash as soon as the potential on the mentioned capacitor, U_c, will overcome the value fixed by the group of three resistors on its positive input. One could also think about this system as the capacitor "measuring the time", and the flashing period being fixed by the group of the three resistors. Two of these resistors are connected to constant potentials (ground and +5 V), but the third resistor is connected to the output of the second comparator IC1A which will have a value depending on the ratio between the reference voltage (G firing threshold) and the U_i characteristic voltage. U_i depends on the amount of light measured by the photo-resistor. When the voltage, U_i, of the photo-resistor is smaller than the firing threshold G, the flashing period determined by the three resistors will be the minimal time-period T_{min}. When $U_i > G$, the flashing period will be T_{max}. This does not mean however that the time between two consecutive flashes can take only these two extreme values. If T_{min} has already expired, and the light intensity suddenly decreases (changing also the
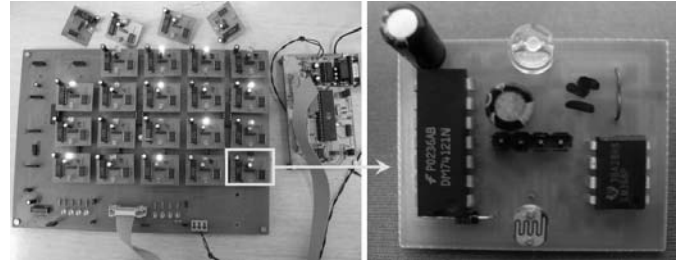

Fig. 1. Experimental setup. The photo on the left shows the "electronic fireflies" (oscillators) placed on the circuit board. The photo on the right shows one oscillator.
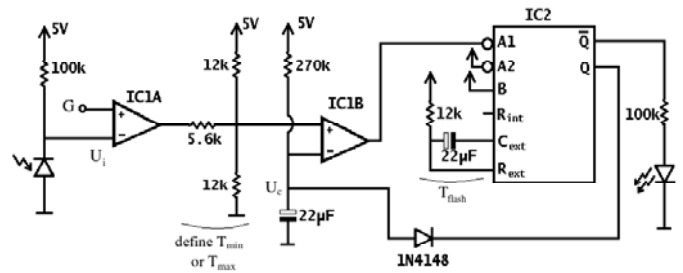

Fig. 2. Circuit diagram of one oscillator.

voltage and consecutively the time-period imposed by the resistors), the flash will be induced in that very moment.

The flash time is determined by the second capacitor together with the 12 KΩ resistor connected to the monostable. The photo-resistor has a relatively low reaction time around 40 ms, while the minimal and maximal period of firing are around 800 ms and 2700 ms. The time of one flash is around 200 ms.

The oscillators are placed on a circuit board in the form of a square lattice (see Fig. 1). The maximal number of oscillators which can be included are 24. A computer interface and program controls the G threshold parameter and allows us to get information automatically about the states of all oscillators. The state of an oscillator is recorded as 0 if the oscillator does not emit light and 1 when the oscillator fires (emits light). Whenever the state of the oscillator system changes, the program writes in a file the corresponding time with a precision of milliseconds and the new states of the units.

To obtain an enhanced global interaction the whole system is placed inside a closed box. The box has mat glass mirror walls to uniformly disperse the light-pulses in the box. A graphical interface allows to visually control the state of the units.

In order to fully understand the behavior of the system one has to accept that the coupling between pairs of oscillators are not exactly of the same strength. Also, one needs to take into account that the characteristic electronic parameters differ slightly (2−10%) among the units and time-like fluctuations or perturbations are also present. These aspect will be discussed in more detail in the next section.

3 Collective behavior

At constant light intensity one unit behaves as a simple stochastic oscillator. Whenever the G threshold is under a given G_c value the oscillator will fire with its minimal period and above G_c with its maximal period. G_c depends of course on the imposed light intensity. Considering more oscillators ($i = 1, \ldots, n$) and by letting them interact, interesting collective behavior appears for a certain range of the G threshold parameter.

Due to the inhibitory nature of the considered interaction, during the firing of oscillator i the characteristic voltages of the others ($U_j, j \neq i$) will decrease. If the G parameter is so small, that under this condition the other oscillators can still fire ($U_j > G$), than all oscillators will fire in an uncorrelated manner. Each of them will be firing at its own T_{min_j} period and the interaction is thus not efficient. In such case no collective behavior can be observed.

Increasing the value of G will make the pulse-like interaction efficient. The oscillators will avoid firing simultaneously and a simple phase-locking phenomenon appears. The pulse of one unit (let us assume i) delays the firing of the others by decreasing their voltages below the threshold: $U_j < G$, $j \neq i$. Due to the tiny differences in the coupling between the pairs (caused for example by different distances) and in the parameters of the electronic elements, the U_j voltages are different. The immediate consequence of this is that the next firing will occur most probably in the oscillator with the highest voltage (counting of course only those oscillators, which are already capable of firing). This oscillator is the one which was influenced the less by the light-pulse of the previous firing. If the total combined time of firing for the n oscillators is smaller than the period T_{max} the result is that after very short time phase-locking appears and a firing chain (with period $T \in [T_{min}, T_{max}]$) will form, each oscillator firing in a well-defined sequence. If the total time of firing of the n oscillators exceeds T_{max}, the firing pattern will be much longer and more complex.

Increasing further and over a limit the G threshold parameter the previously discussed weak synchronization (phase-locking) disappears. In this case the voltages of all oscillators are much smaller than the threshold value $U_i < G$, so the firing of a unit can not influence the others. All oscillators will fire with their own T_{max_i} period and no interesting collective behavior is observed. Again, the interaction is not efficient.

The collective behavior of the system can be easily analyzed by plotting a kind of *phase-histogram* for the oscillator ensemble. Choosing a reference oscillator, the relative phases of all the others are defined by measuring the time difference between their pulse and the last pulse emitted by the reference oscillator. Studying these time-delays during a longer time period a histogram is constructed for their distribution. This histogram shows how frequently a given time-delay occurred and gives thus a hint whether a constant firing pattern is formed or not.

Experimental and computer simulated results for the phase-histogram confirm the above presented scenario of the collective behavior. As an example, in Figure 3, results obtained on a relatively small system with $n = 5$ oscillators are shown. In the first column of Figure 3 (Figs. 3a–3d), experimental results for four different values of the G threshold are plotted. For a small threshold parameter ($G = 500$ mV), no self-organization appears (Fig. 3a). Due to the fact that the characteristic time-periods of the oscillators are slightly different, almost all values will occur with the same probability in the phase-histogram. Beginning with $G = 1300$ mV a kind of order begins to emerge, and a trend towards the self-organization of the oscillator pulses is observed (e.g. Fig. 3b for $G = 2000$ mV). In the neighborhood of $G = 3000$ mV threshold value (Fig. 3c) clear phase-locking appears. One can observe that a stable firing pattern has formed, each oscillator has an almost exact phase relative to the reference oscillator. For an even higher value (e.g. $G = 4200$ mV), disorder sets in again, phase-locking disappears and all oscillators fire independently with their own maximal period (Fig. 3d).

In the second column of Figure 3 we present the corresponding simulation results. In simulations the parameters of the oscillators are defined as following: the average minimal time period is $T_{min_i} = 900$ ms, the average maximal period $T_{max_i} = 2700$ ms, and the average flashing time $T_{flash} = 200$ ms. For an easier comparison, the values are chosen to be similar with the real experimental data. We considered a uniform distribution of the oscillators parameter around these average values using a ± 50 ms interval for T_{min} and T_{max} and a ± 20 ms interval for T_{flash}. One could argue of course that a Gaussian distribution would be much more appropriate, but given the fact that we simulate here relatively small number of oscillators the exact statistics is irrelevant. Considering some deviations from the average is however important in order to reproduce the collective behavior of the system.

If the oscillators are considered to be identical (i.e. the noise in the system is so small that the differences between the periods are smaller than the simulation time-step or the reaction time of the photo-resistor in the experiments), than usually phase-synchronization can be observed in the whole, or sometimes parts of the system. For example, starting from the same initial condition, the oscillators will reach T_{min} exactly at the same time, and flashing occurs at the same time. If we do not start from the same initial condition, than the first oscillator reaching T_{min} will fire first, it will delay the flash of the others, but after the firing of this first one ends, again all the others will fire simultaneously. And this pattern remains stable, assuming the periods of all are the same, and do not fluctuate in time.

An uncorrelated time-like noise is also considered. This will randomly shift the T_{min}, T_{max} and T_{flash} periods of each oscillator at each cycle. Again, a uniform distribution on a ± 20 ms interval was considered. If one introduces random differences between the periods of the oscillators, but there are no fluctuations in time, phase-locking already appears. The oscillators do not reach their T_{min} simultaneously, and the first firing always delays the
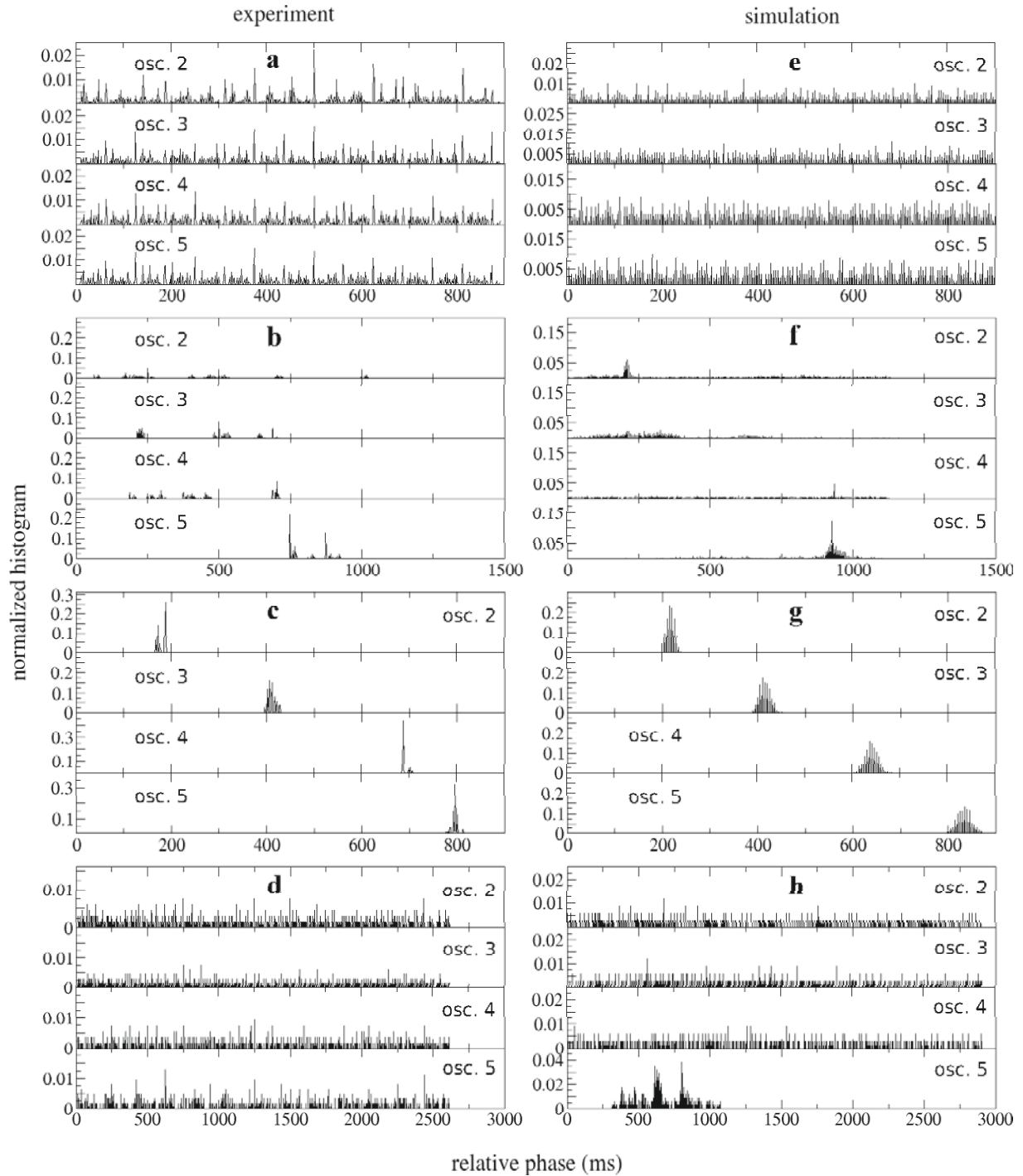
experiment simulation

Fig. 3. Relative phase histogram for $n = 5$ oscillators. Experimental results are in the first column, and the corresponding simulation results are in the second column. (a) and (e) are for $G = 500$ mV; (b) and (f) are for $G = 2000$ mV; (c) and (g) are for $G = 3000$ mV, and (d) and (h) are for $G = 4200$ mV.

other ones. It can still happen that the difference between two oscillators is too small and they fire simultaneously. The phase-locking scenario observed in the experiments is obtained when also time-like fluctuations are added. For reproducing the experimentally observed results the noise on T_{min} or T_{flash} is the most important ingredient. T_{max} is less important, because in the region for G where

phase-locking is observed, the oscillators reach their T_{max} period only seldom. The noise on the voltages U_i or G are also important: because of these differences and the nonlinear characteristics of the photo-resistor, the oscillators will never reach their threshold at the same time. However in the considered simplified model we do not simulate the nonlinear behavior of the photo-resistor, so the
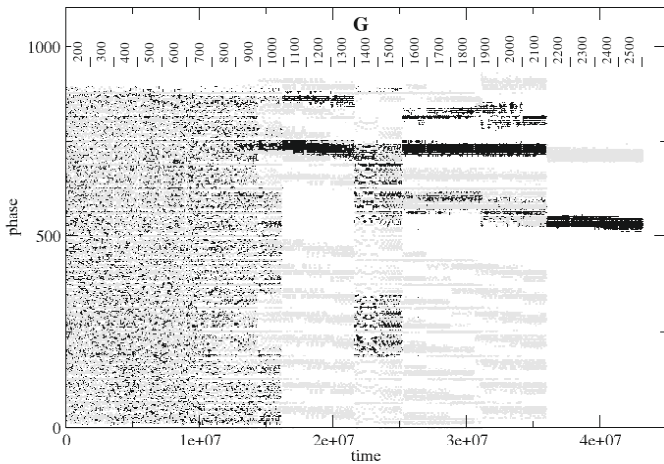
Fig. 4. Stroboscopic time-evolution map for $n = 3$ oscillators. The G threshold value changes in time as indicated on the top of the figure. On the horizontal axes we represent the time and on the vertical axes the time delay (phase in ms) of the firing oscillator relative to the last firing of the reference oscillator. Firing of oscillator 1 is plotted with black pixels, firing of oscillator 2 with gray pixels and oscillator 3 is the reference oscillator.

Fig. 5. Stroboscopic time-evolution map for $n = 3$ oscillators while the G threshold value changes in time. (a) and (b) are for two different experiments with the same oscillators placed in the same position on the circuit board.

noise on these values has an unimportant role. The fact that time-like fluctuations do not influence in a qualitative manner the obtained statistics proves that this behavior is stable against time-like fluctuations in the characteristics of the electronic elements or small external perturbations.

The characteristic voltages of the oscillators are set to be in the interval 4100 ± 100 mV in dark, 2100 ± 100 mV when one single LED is flashing and 1050 ± 100 mV when two LEDs are flashing simultaneously. Whenever k LEDs are simultaneously flashing the characteristic voltages of the others are considered to be $2100/k \pm 100$ mV, however for $n = 5$ oscillators only very rarely happens that more than two oscillators are simultaneously firing. The above values were chosen to approximately match the experimental ones. Differences in the strength of the coupling between pairs of oscillators are however neglected. Using these parameters, it is assumed that each oscillator can flash whenever its voltage exceeds the threshold G. The flashing cannot occur earlier than T_{min_i} or later than T_{max_i} relatively to its last firing. In Figures 3e–3h, the simulated phase-histograms of the oscillators are plotted and compared with the corresponding experimental data. The observed experimental results, including the non-trivial synchronization (phase-locking), were successfully reproduced.

Another possibility for characterizing the collective behavior of the system and to give also a better picture on the time-evolution is to consider the stroboscopic time evolution map used in references [15,16]. In this representation one chooses a reference oscillator and then plots the firing events of the others as points on a graph. In this graph on the horizontal axes one represents the evolution time and on the vertical axes we have the time delay (phase) of the firing oscillator relative to the last
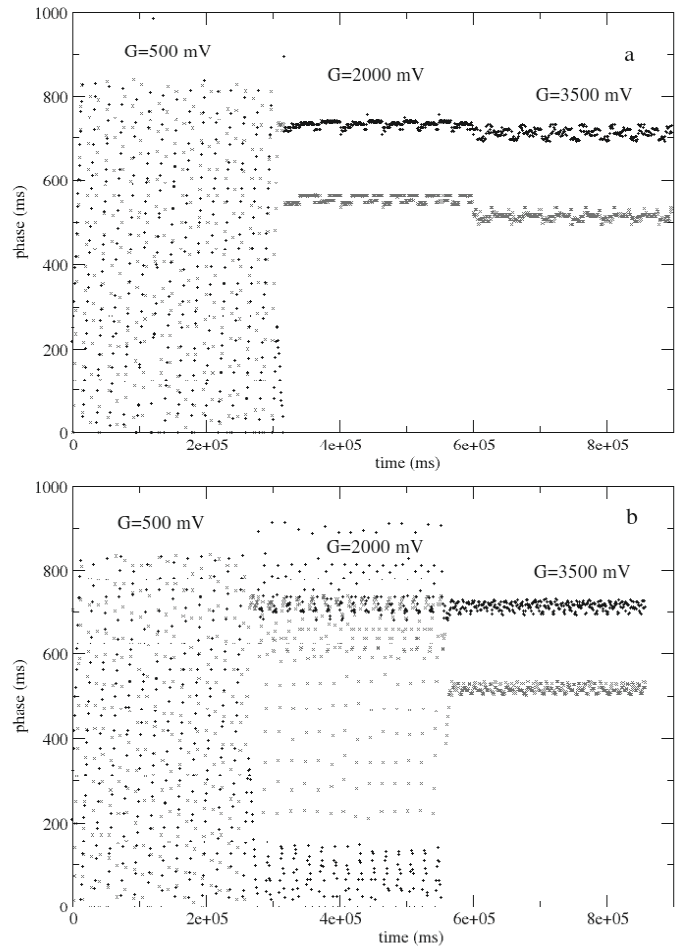
firing of the reference oscillator. If different oscillators are plotted with different colors, the dynamics of the system is completely revealed. This representation offers also the possibility to follow dynamically what happens when the G threshold parameter is changed. Without color coding it is possible to present only the case of $n = 3$ oscillators (the firing of one oscillator with black pixels and the firing of the second one with a gray pixel, the third oscillator being the reference one). The result of a complete experiment where the G threshold parameter is changed in time with a $\Delta G = 100$ mV step is plotted in Figure 4.

From Figure 4 it is nicely observable how phase-locking emerges as the threshold value is varied. It is also observable the complex flashing patterns before the clear phase-locking appears.

Using this stroboscopic representation we can also prove that the whole phase-locking scenario is sensible to the initial conditions. By repeating the whole experiment with the same oscillators placed in the same positions on the circuit board the scenario can completely change depending on the uncontrolled starting phases of the oscillators. For $n = 3$ oscillators this is illustrated in Figure 5.
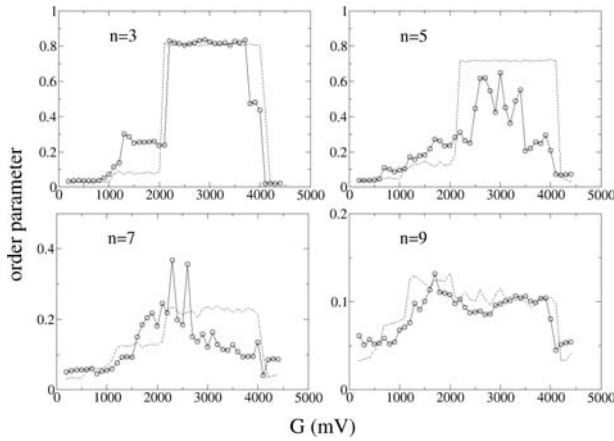
Fig. 6. Order parameters calculated from experimental (circles) and simulation (dashed line) results plotted as a function of the G threshold. Systems with $n = 3, 5, 7, 9$ oscillators are considered.

This suggests also that the firing patterns are sensible on the position of the oscillators on the circuit board.

It is also possible to define an order-parameter that characterizes the observed synchronization level. Our method for calculating this is the following:

1) A reference oscillator k is chosen and the phases of all oscillators are calculated relative to this oscillator.

2) Let $h_i(f)$ denote the value of the normalized phase-histogram for oscillator i ($i = 1, \ldots, n, i \neq k$) corresponding to phase difference (time difference) value f. Since we have a normalized histogram, $h_i(f) \in [0, 1]$ gives the occurrence probability of phase difference value f during the measurement ($\sum_f h_i(f) = 1$).

3) A window of width a is defined (we have chosen $a = 30$ ms). Shifting the window with $\Delta f = 1$ ms step, for each value of f the sum $H_i(f) = \sum_{j=f-a/2}^{f+a/2} h_i(j)$ is calculated for each oscillator i.

4) Let r_k denote the difference between the maximum and minimum value of $H_i(f)$ averaged over all oscillators: $r_k = \frac{1}{n-1} \sum_{i=1,(i \neq k)}^{n} \max(H_i) - \min(H_i)$.

5) Items 1–4 are repeated considering each oscillator in the system as reference oscillator.
   Finally, an averaging is performed over all the obtained r_k values ($k = 1, \ldots, n$). The final order parameter is calculated thus as $r = \langle r_k \rangle_k$. Averaging as a function of the reference oscillator is beneficial in order to get a smoother curve when only partial phase locking is detected (Fig. 3b). In such cases the phase-diagrams are very sensible on the choice of the reference oscillator.

In Figure 6 the r order parameter is plotted as a function of the G threshold value. Systems with $n = 3, 5, 7$ and 9 oscillators are considered. Experimental (circles) and simulation results (dashed line) are again in good agreement. The figure also illustrates that for an intermediate G interval value phase-locking appears. This weak synchronization is better (r is bigger) when there are less units in the system. One obvious reason for this is that by increasing n the total time of firing of the oscillators will increase and slowly exceed the value T_{max}. As a result of this the firing pattern will change from a simple "firing chain" to a much longer and more complicated pattern, decreasing the value of the order parameter.

From Figure 6 it is also observable that the experimental results show more intensive fluctuations. The reason for this is probably the complex noise present in the system.

4 Conclusion

A system of electronic oscillators communicating through light-pulses was studied. The units were designed to optimize the average light intensity of the emitted light-pulses, and no direct driving force favoring synchronization was considered. Although our experiments focused on relatively small systems (up to 24 oscillators) interesting and rich collective behavior was observed. As a nontrivial result it was found that the light intensity optimization induced a partial phase-locking through an inhibitory coupling for a certain interval of the controllable threshold parameter. This weak synchronization was realized by complex flashing patterns of the units. We believe that this study inspires further interesting research projects in which separately programmable oscillators will be studied with various interaction rules.

Studying the behavior of programmable interacting units is important from the perspectives of information technology, because Moore's law is expected to be continued by increasing the number of processor cores. The algorithms for efficiently coordinating the units are still missing. When studying the behavior of these kind of systems, one of our long-term question is what useful functions could be solved with these interacting units. We can not answer this question thinking in the classical way: here we have the problem, how can we solve it? We have to think inversely: this is the behavior of a complex system we have and we should then find the problem which can be solved using the nontrivial response of this system. For elaborating new computational paradigms the behavior of simple systems like this could be of great importance.

Our system is in many sense similar to the presently developed Cellular Neural/Nonlinear Network (CNN) computers [19]. CNN computers try to imitate some basic principles of our nervous system (specially the retina): (i) several thousands of microprocessors (cells, neurons) are placed on a single chip locally interacting with each other and working totally in parallel; (ii) the states are usually described with analog values; and (iii) the evolution of the system is continuous in time. These CNN chips are generally used and developed for fast image processing applications, the new cellular visual microprocessor Q-Eye [20] has for example 25 000 processors. Our system is based on similar ideas, and the type of coupling realized by communication with light-pulses could be a useful idea even

in hardware projects, because it resolves wiring problems and global coupling can be achieved.

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