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Spring-block model for a single-lane highway traffic

Research Article

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Abstract: A simple one-dimensional spring-block chain with asymmetric interactions is considered to model an idealized single-lane highway traffic. The main elements of the system are blocks (modeling cars), springs with unidirectional interactions (modeling distance-keeping interactions between neighbors), static and kinetic friction (modeling inertia of drivers and cars) and spatiotemporal disorder in the values of these friction forces (modeling differences in the driving attitudes). The traveling chain of cars correspond to the dragged spring-block system. Contrary to most of the studies in the field of highway traffic here we focus on a measure characteristic for one car in the row. Our statistical analysis for the spring-block chain predicts a non-trivial and rich complex behavior. As a function of the disorder level in the system a dynamic phase-transition is observed. For low disorder levels uncorrelated slidings of blocks are revealed while for high disorder levels correlated avalanches dominates.

1. Introduction

Road traffic is a non-linear complex phenomenon which strongly affects our everyday life. Within this phenomenon various forms of collective behavior may be detected. The simplest possible form of traffic is the one on a single highway lane. The motion of the queue in this simple form of traffic is primarily governed by the leading car and the statistics of driving attitudes. This simple situation becomes already quite complex if the first car is moving slowly and the differences between the driving attitudes are substantial. In such situations the queue will

evolve non-continuously in avalanches of different sizes, and jams of different magnitudes will continuously appear. Unfortunately, such situations are common in our everyday life, so understanding and modeling it is important in order to optimize our society.

Traffic studies tend to discover fundamental rules in different kind of transport systems that are essential for our social and economic efficiency. Accordingly, in this field many theoretical models have been developed. The study of traffic begins early in 1935 with the pioneering work of Greenshields [1]. In 1955 Lighthill and Whitham published the oldest and most popular macroscopic model based on the theory of fluid dynamics [2]. After these early works, and motivated by the explosive increase in road traffic, an avalanche of publications in leading international journals began. These models can be classified into four cat-

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egories: microscopic models, macroscopic models, cellular automata models and non-traditional models. Detailed description and analysis of such traffic models can be found in many review articles [3–6]. More recently, the nonlinear effects of small perturbations have encouraged the development of stochastic models [7] for the highway traffic. A complete review of these traffic studies can be found in the recent work of Nagatani [8], or in Kerner's book [9]. Despite of many studies in the field, due to the complexity of the problem, there are several phenomena which are still not well understood.

In order to get familiar with the complexity of the traffic and with the typical non-linear avalanche-like phenomena that can easily occur in highway traffic, let us imagine that we are driving on a highway in the middle of a car queue. The traffic is normal with a laminar flow structure shown in Figure 1a. Suddenly, the driver of the car located in front of us sees something and slows down a bit. To avoid collision, we should slow down only a bit, we should only reduce the foot pressure on the gas pedal. But, being concentrated to the car audio system instead we observe a little later the event and consequently we are forced to step the brakes. Seeing our braking lights, the driver behind us suddenly brakes and stops at the lane. A small event caused, thus, a substantial change in the traffic flow pattern. A wave consisting of cars with higher density forms and the laminar flow turns into a start-stop-start-stop motion of cars caused by temporary traffic jams that appear and propagate as sketched in Figure 1b.
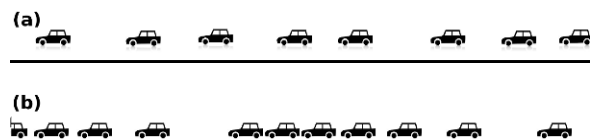

Figure 1. Traffic with normal, laminar flow structure (a) and traffic with jams (b).

Such situations are investigated in the present work by means of computer simulations. A simple spring-block type model with asymmetric interactions is used to model the phenomenon and the effect of parameters having major influences on the dynamics of the system will be investigated. For some model parameters, interesting collective behavior is found and it is statistically studied. Up to our knowledge real experimental data describing the dynamics of cars in such idealized situation (cars restricted to one lane only and moving with a constant average flux) is not available. Only one qualitative experiment focusing on the formation of the jams exists without any statistical data collection [10]. Therefore, at this stage the study has to limit itself on pure modeling, and the test of the

results rely simply on our everyday experience. If in simulations a simulated quantity proves to be relevant, then targeted experiments can be then planned, and both the assumptions of the model and the predicted collective behavior can be later verified. In this sense the present work intends to motivate also further experimental works.

2. The spring-block model

A model family with broad interdisciplinary applications is the spring-block type models. Previously we have used them successfully to study complex systems which have shown self-organization phenomena. This model family was introduced in 1967 by R. Burridge and L. Knopoff [11] to explain the empirical law of Guttenberg and Richter [12] on the size distribution of earthquakes.

This earthquake model consists of simple elements: blocks that can slide with friction on a horizontal plane connected in a lattice-like topology by springs. In the one-dimensional version of the model, the involved tectonic plates are modeled by two surfaces and their relative motion is governed by the sliding of the spring-block system on the lower surface. The upper surface (to which each block is connected by a spring) is dragged with a constant velocity. Sliding of the blocks is realized by avalanches, as they are following the motion of the upper plate. These avalanches correspond to earthquakes and the energies dissipated by friction show power law type distribution as in case of the Guttenberg-Richter law.

Subsequently, due to the spectacular development of computers and computational methods, this simple model proved to be very useful in describing many phenomena in different areas of science. Most of the collective phenomena that occur on mesoscopic scale in solid materials can be modeled by spring-block models. The model is almost always usable when one has to deal with avalanches of different nature, complex dynamics or structure formation by collective behavior. Recently, this model has been applied successfully to explain the formation of structures obtained in drying granular materials in contact with a surface [14], to study the fractures with curved topologies in wet granular materials [15], to understand the formation of self-organized nanostructures produced by capillary effects [16–18], to study the magnetization processes and Barkhausen noise [19] and to describe glass fragmentation [20]. Based on the previous examples, one can conclude that the spring-block type models are fully applicable to the study of complex collective behavior in systems of different nature.

The spring-block model for idealized single lane highway traffic can be imagined as a chain of blocks (see Fig-
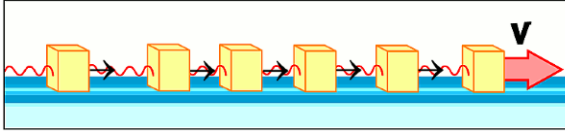
Figure 2. Sketch of the spring-block chain considered here to model the idealized single lane highway traffic.

ure 2), modeling cars, connected by springs which represent some distance keeping interactions between them. This interaction represents the will of drivers to keep a certain distance from the car ahead. Therefore, in order to be realistic, these springs cannot be real, bi-directional mechanical springs because in case of traffic without accidents this distance keeping interaction acts only on the car in the back. The front car is never pulled back or pushed ahead by the car from behind unless there is a collision. Accordingly, the action–reaction principle is violated and, in this sense, this spring-block system cannot be a considered as a typical mechanical system.

Another important ingredient that has to be incorporated in the model is the disorder in the system generated by the driving style of drivers. From our point of view, this should be the drivers' inertia which indicates how quickly the driver can react to a certain event or how quickly can follow the velocity change of the car ahead. This is introduced via friction forces between the blocks and the plane on which they slide. These friction forces are generated randomly and independently for each new position of each of the blocks. As a first approximation we consider a normal distribution with a fixed mean F_m and standard deviation σ. As a result of this, the disorder introduced is both spatial and temporal. Spatial disorder means differences in driving styles while temporal disorder means fluctuations of driving attitudes in time.

Moreover, in analogy with real mechanical systems, static and kinetic friction forces are considered denoted by F_s and F_k, respectively. As in usual classical mechanics systems, the static force is considered to be greater than the kinetic one, and for simplicity in our model their ratio is kept constant

$$f = \frac{F_k}{F_s}.$$

Here we have to note that for all the presented simulations $f = 0.8$ has been used. Of course, this parameter is an adjustable one and its influence on the dynamics of the system is planned to be investigated later.

Regarding the distribution of the friction forces one more comment needs to be added: when static friction forces are randomly updated in each new position of the blocks, the kinetic friction force is updated, too. As a result, both

the kinetic and static friction forces acting on blocks will fluctuate in time and in space.

The motion of the block queue is simulated in discrete simulation steps. For each step it corresponds the same time length $dt = 1$, fixing the unit for simulation time and the possibility to handle easily the stochasticity in the equations of motion. Due to this choice we do not have possibility to define a velocity for the first block, and for us a given mean value of static friction F_m and spring constant k implicitly corresponds to a fixed drag velocity. Accordingly, the motion of the queue is governed by the drag step (the movement of the first block) which is kept constant in time. In each unit of time the first block moves ahead in steps of length d_0. In the present simulations for the spring-block row a step limit of $d_{max} = 1$ is imposed. This represents a kind of speed–limit for the units in the system. It is important to realize that by changing the drag step value we do not change the velocity of the first block. Instead of this we consider either a finer (for small d_0 values) or a rough (for large d_0 values) approximation of the continuous dynamics.

The length unit in the simulation is defined by the length of a single block $L = 1$. In order to simulate the simplest dynamics without accidents, a minimum distance between blocks $d_{min} = 0.3$ is imposed. This distance is considered to be also the equilibrium length l_0 of the asymmetric springs.

The motion of each block is governed by the total force acting on it. The force unit in the model has been selected by considering that all springs have the spring constant $k = 1$. Unit elastic force acts on the block from behind when the distance between the two blocks is $(d_{min} + 1)$ expressed in simulation length units.

With this force unit we fix the mean value of the normal distribution for friction forces $F_m = 4$, and in the following the σ parameter is considered to be the main parameter governing the disorder in the system.

Therefore, in this study, the model will have two free parameters: the drag step d_0 of the first block and the disorder level in the static/kinetic friction σ. All other model parameters have been fixed as $d_{min} = 0.3$, $d_{max} = 1.0$ and $F_m = 4.0$ and they will not be explicitly noted in our later discussion.

Blocks are labeled after their ordinal number in the row so that the dragged block (the first in the queue) has label 1, the next one label 2, and so on until the number of blocks in the queue N. Their position will be noted by $x^{(i)}$, where $i = \overline{1, N}$.

The simulation follows the typical steps of a simplified molecular dynamics simulation summarized bellow.

I) All blocks are visited and the spring force F_{spring} act–

ing on each block is calculated

$$F_{spring}^{(i)}(t) = k\left[x^{(i-1)}(t-1) - x^{(i)}(t-1) - L - l_0\right]. \quad (1)$$

If the block is at rest (its previous displacement is 0) the spring force is compared to the static friction. If $F_{spring}^{(i)}(t) \leqslant F_s^{(i)}(t)$ the static friction equals the spring force and the total force acting on the block will be

$$F_t^{(i)} = 0, \quad (2)$$

and the block will remain at rest in this step. On the contrary, if $F_{spring}^{(i)}(t) < F_s^{(i)}(t)$ the block will start to move and the kinetic friction force is added to the spring force and

$$F_t^{(i)} = F_{spring}^{(i)} - F_k^{(i)}. \quad (3)$$

The total force is similarly calculated, when the block is not at rest (its previous displacement differs from 0).

II) All blocks are visited and, based on forces $F_t^{(i)}$ calculated in simulation step I, their displacements $\Delta x^{(i)}$ are calculated. The displacement of the first block is $\Delta x^{(1)} = d_0$ which represents the constant drag step of the first block. For the rest of the blocks $i = \overline{2, N}$ the displacement is calculated by using the equation of classical mechanics with $dt = 1$

$$\Delta x^{(i)}(t) = \Delta x^{(i)}(t-1) + AF^{(i)}(t), \quad (4)$$

where $A = \frac{1}{2m}$, m being the mass of a block. In our simulations this constant is chosen to be $A = 1$ which fixes the mass of blocks to $m = 0.5$ expressed in simulation units.

III) For the calculated displacements, the block step limit d_{max} is applied which means that all displacement values grater than d_{max} are set to be d_{max}.

IV) All blocks are visited following their ordinal numbers and their new positions are calculated

$$x^{(i)}(t) = x^{(i)}(t-1) + \Delta x^{(i)}(t). \quad (5)$$

Here the only restriction is that the minimum distance d_{min} between blocks has to be respected.

V) Quantities of interest for the selected block are detected and stored for later analysis.

Steps I-V are repeated until enough data is collected for the statistical analysis.

3. Results of modeling

The dynamics of the system is illustrated with a short queue of 50 blocks in Figure 3. The first block is moving to the right with small drag step $d_0 = 0.01$. The snapshots are taken on the time steps indicated in the figure. It has to be noted that between time steps 1000 and 1100 the propagation of an avalanche through the whole system is observable. Based on a visual comparison with the real experiment of Sugiyama [10] the model system dynamics seems to be realistic.
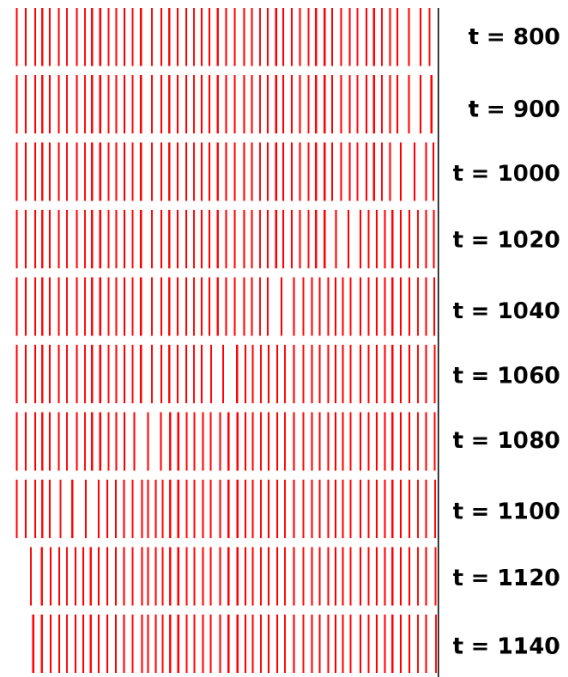

Figure 3. Time evolution of a queue with 50 blocks.

In real traffic conditions the experimental data are usually collected by detectors located at fixed places. Accordingly, the most studied quantity related to traffic flow is the vehicle flow (the number of passing cars in a given time interval). There are also some other related and measured quantities like time headways, time clearances and time occupancy (for the definition of these measures one can consult the review work [4]). These quantities are, however relevant for the whole system and not focusing on a single car in the row. In the present work we turn the question around and we focus on what a driver stuck in the middle of the queue is experiencing.

In such view, in our one-lane traffic model the stop-time distribution $g(\tau)$ is a relevant measure. This stop-time distribution describes the distribution of time intervals during which a given driver is not moving. In other words,

this distribution function $g(\tau)$ determines the probability that the rest time of the car is τ. Alternatively, the cumulative distribution function $g_>(\tau)$ may be defined, which gives us the probability that the stop-time of a car is bigger than τ. Based on the distribution function the mean stop-time and the stop-time standard deviation may be calculated. In real traffic systems this stop-time statistics characterizes the flow of the traffic from the point of view of a single driver. It also gives a hint on how predictable the traffic flow is for the driver. For instance, if the stop-time distribution has a narrow peak, drivers can predict how much time they are blocked in the traffic. On the contrary, for a power-law type long tail distribution the waiting times are unpredictable.

First, let us investigate how the position of a block in the queue is influencing its stop-time distribution. In Figure 4 the stop-time distributions of different blocks in the row are presented for a fixed small drag step $d_0 = 0.01$ and disorder in the static friction forces $\sigma = 0.4$. In these simulations $N = 1000$ blocks have been used and the distribution function is constructed using the statistics of 100000 stop-times. From this results it can be learned that after a certain number of blocks (transient distance), the cumulative stop-time distribution converges to the same function. Therefore, in order to be sure that the average behavior of a single block is studied, in our later investigations a block after this transient distance has to be selected. The form of the single block stop-time distribution function will be discussed later.
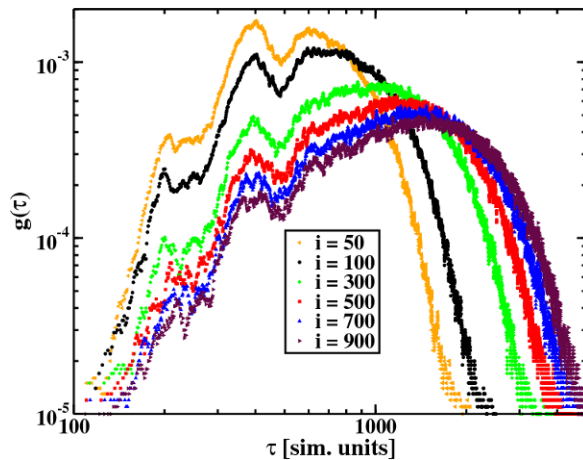

Figure 4. Stop-time distributions for different block positions i. The plotted results are obtained from a statistics of 100000 stop-times.

the middle of the queue $i = 500$ and several disorder levels $\sigma = 0.1, 0.5, 1.0, 1.5, 2.0$ are imposed. The results are plotted in Figure 5. As one may observe, except for high drag step values the average stop-time $\langle\tau\rangle$ and the standard deviation of stop-times $\Delta(\tau)$ scale with the drag step following a $\frac{1}{d_0}$ functional dependence. In order to understand these scalings we have to remind ourselves again, that the drag step is not equivalent with a drag velocity, and all this is a result of the fact that the simulation time-step was fixed a priori to $dt = 1$. This means that if one wants to get close to a realistic, continuum dynamics, the drag step has to be lowered to infinitesimally small values. Changing the drag step is in fact equivalent to a change in the time-length dt of a simulation step and thus influences only the real time of a simulation step, and implicitly everything else which is measured in time units. Therefore, this scaling suggests only that the simulations are performed in the right continuum limit. Based on our results we find that this limit is reached if drag steps smaller than 0.1 are selected.
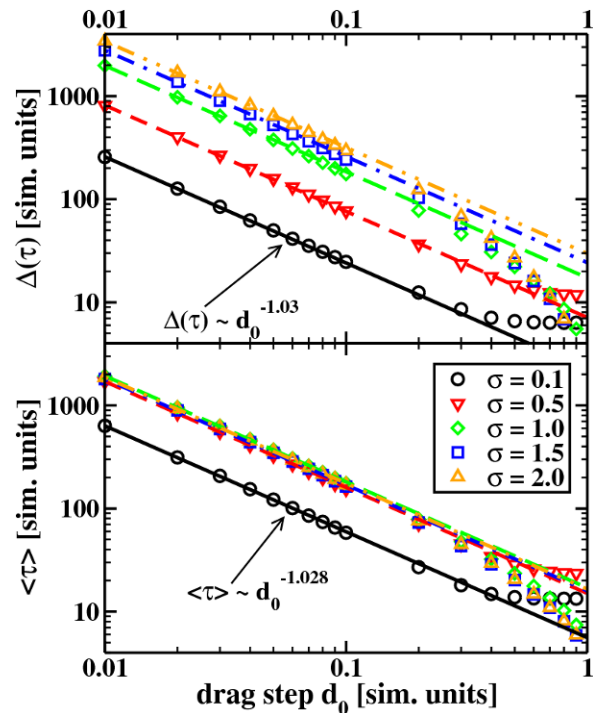

Figure 5. Stop-time averages (bottom) and standard deviations (top) depending on the drag step calculated for the 500th block in a queue of 1000 blocks. These quantities are calculated from 100000 stop-times.

Another more illuminating analysis may be performed if one looks at the stop-time averages and standard deviations. First, their dependence on the drag step is investigated. The position of the studied block is fixed at

Concerning the dynamics of the studied system interesting and non trivial results may be obtained if one looks at the influence of the friction force disorder level σ on the stop-time averages and standard deviations. In this sense

a series of simulations have been performed for several drag step values between 0.01 and 0.1. The position of the studied block is fixed at the middle of the queue ($i = 500$), and the averages and standard deviations are calculated for 100000 stop-times. The simulation results are plotted on Figure 6.
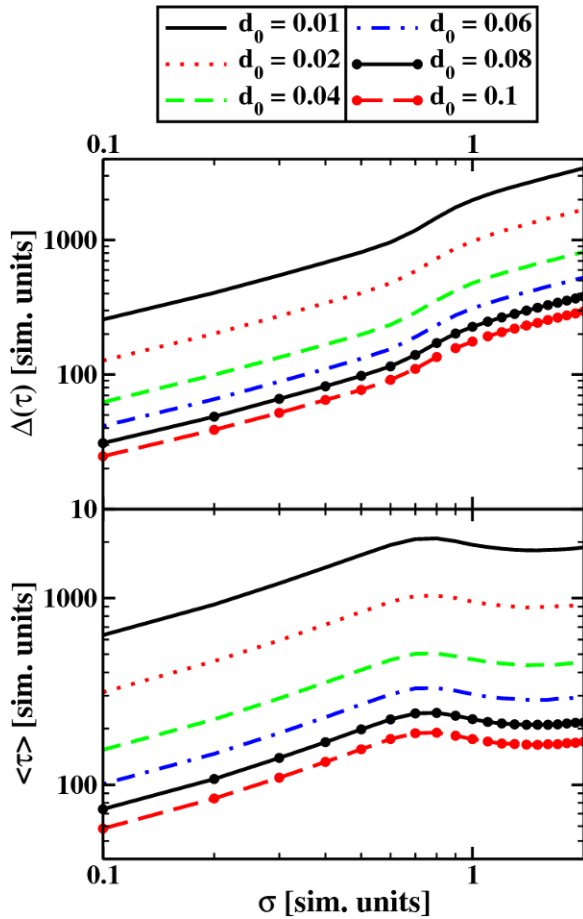

Figure 6. Stop-time averages (bottom) and standard deviations (top) depending on the disorder in static friction calculated for the 500th block in a queue of 1000 blocks. These quantities are calculated from 100000 stop-times.

On the one hand, as it is expected based on our previous considerations, the statistical behavior of the system for different and small enough drag steps is the same. On the other hand, for all the curves, at a certain disorder level ($\sigma \simeq 0.7$) a maximum in the stop-time average may be detected. This result shows us that in our simple traffic system there is a "worst" disorder level in driving attitudes (modeled by friction forces) for which the average

stop-time of a block in the row is maximum. Moreover, in the case of the stop-times standard deviation as a function of the mean disorder in the friction force value, at this "worst" disorder level an inflection point may be detected.

Therefore, based on these observations we can conclude that this simple spring-block system apparently exhibits two different types of behavior as a function of the disorder level in the system. At low disorder levels in the friction forces the average stop-time scales with the disorder level. This scaling law disappears however for higher disorder levels. Since this change in the system dynamics appears for a certain disorder level, we consider it as a *disorder induced phase transition* similar with the one recently obtained in the spring-block model of magnetization phenomena [19]. This phase transition is a higher-order one, because a peak only in the derivative of the stop-time's standard deviation is detectable.

In order to take a closer look at the nature of this athermal phase transition detected in our spring-block system stop-time distributions of a single block are analyzed. Three disorder levels $\sigma = 0.4, 0.6, 0.9$ are selected which correspond to values lower than, close to, and higher than the critical disorder level.

The distribution functions are constructed from 100000 simulated stop-times and they are plotted in the left panel of Figure 7 for different drag steps of $d_0 = 0.01, 0.05, 0.1$. As it is immediately observable, near the critical disorder level (central graph on the left panel of Figure 7) there are two peaks with the same magnitude in the stop-time distribution function best visible for drag step $d_0 = 0.01$. At lower disorder levels (top graph on left panel of Figure 7) the first peak disappears and the stop-time distribution of long stop-times will be close to a normal distribution. This may be explained if we suppose that in this region the motion of blocks is independent to each other and it is mainly influenced by friction forces generated randomly from a normal distribution. On the contrary, at higher disorder levels (bottom graph on left panel of Figure 7) the first peak, which has an exponential nature, will have the most important contribution to the stop-time distribution function suggesting stops that are induced by collective effects. It is extremely important to note that these exponential curves fall into a single one in the case of different drag steps. Taking into account our previous results concerning the $\langle \tau \rangle \sim \frac{1}{d_0}$ scaling of average stop-times, this observation gives us an evidence that this part of the stop-time distribution is not depending on the stop-time averages and it is caused by an avalanche-like collective motion of blocks.
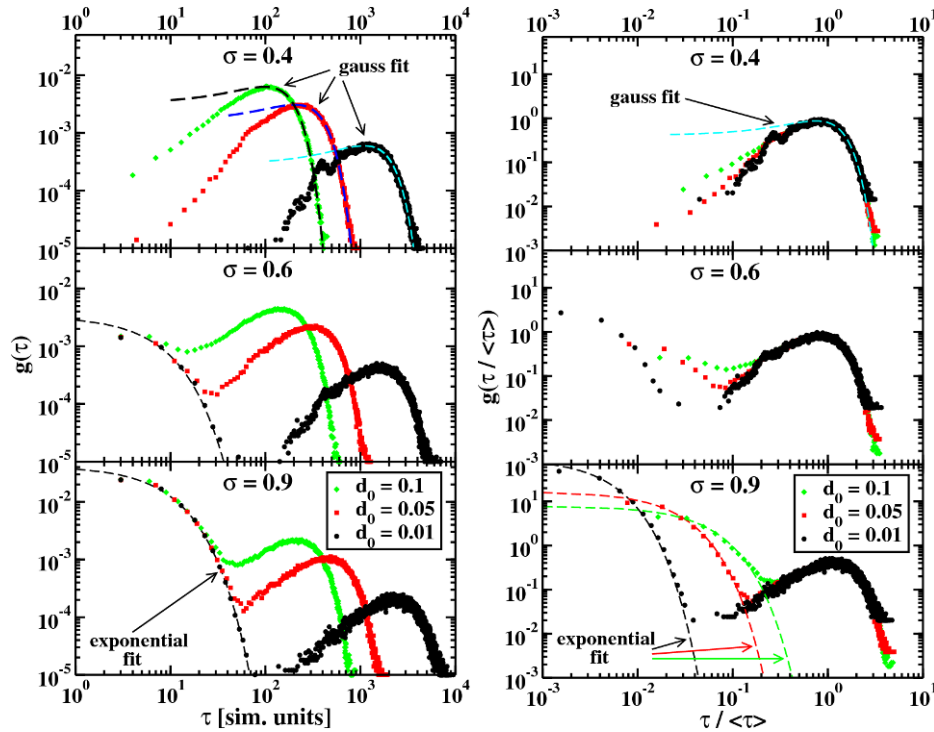
Figure 7. Stop-time distributions (left) and normalized stop-time distributions (right) of a single block in the middle of a queue for different disorder levels of the friction force.

The same data may be analyzed from another point of view if the distribution of normalized stop-times $\frac{\tau}{\langle\tau\rangle}$ is constructed. These curves are plotted on the right panel of Figure 7. It is immediately observable that the Gaussian parts of distribution functions fall into a single curve. This confirms our previous presumption that the block motions at small disorder levels are mainly independent to each other which causes that the resultant distribution of normalized stop-times to be the same.

Therefore, our simulation results show that the studied spring-block system may exhibit non-trivial, critical behavior. Through the detected disorder induced higher-order phase transition the independent block motions will be organized to a highly correlated avalanche-like collective motion of the queue.

4. Conclusions

In summary, in the present paper a simple spring-block type model with asymmetric interactions has been used to model the simplest possible, idealized form of traffic which happens on one single highway lane with a constant average flux of cars. The spring-block chain dragged by constant steps of the leading block predicts non-trivial, complex behavior even for this simple form of traffic. Our investigations have been performed by a series of computer simulations targeting the effect of the relevant parameters on the model dynamics. The study focused on a measure which is relevant for one driver (block) in the queue: the distribution of the rest times. The majority of earlier studies considered however the vehicle flow at fixed points. As a result, it was concluded that at low disorder levels in friction forces the dynamics is governed by uncorrelated sliding of the blocks. On the contrary, at higher disorder levels the dynamics of the block sliding self-organizes into an avalanche-like motion of the blocks. The transition between these domains is realized through a disorder induced phase transition whose effects are detectable in stop-time averages and standard deviations, too.

Acknowledgments

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