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OPTIMIZATION INDUCED COLLECTIVE BEHAVIOR IN A SYSTEM OF FLASHING OSCILLATORS

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An array of pulse-emitting oscillators capable of emerging collective behavior is investigated by computer simulations and through a simple experimental setup. The oscillators emit pulse-like signals and detect the signal emitted by the others. They have stochastically fluctuating periods and can operate in two different modes, one with a short output pulse and one with a longer one. The switching between modes is governed by a simple optimization rule: whenever the total output in the system is lower than a desired f^* threshold level they emit long pulses and when the output is higher than f^* they emit short-length pulses. This simple dynamical rule optimizes the average output level in the system around the f^* value and acts as a coupling between the units. As a side-effect of this simple dynamics complex collective behavior appears. In spite of the fact that there is no direct phase-minimizing interaction between the units, for a certain f^* interval the pulses of the oscillators synchronize. Synchronization appears and disappears abruptly as a function of the f^* threshold parameter, suggesting a dynamic phase-transition. In the synchronized phase the collective output of the system has a better periodicity than the oscillators individually. A simple experimental setup with flashing multimode oscillators is built. For a given range of the threshold parameter the experimental setup reproduces the theoretically predicted synchronization.

Keywords: Synchronization; multimode oscillators; phase-transition; collective behavior.

1. Introduction

Physics has dealt with collective behavior much earlier than when the expression collective behavior first began to appear. Systems with a big number of interacting components that can together produce emerging effects on a larger scale are familiar to physicists. Synchronization [Strogatz, 2003] is the most well-known form of collective behavior in science, and maybe it was first studied by the physics community. If the legend is true, the Dutch physicist Christian Huygens was the first to study scientifically this phenomenon following the dynamics of two pendulum clocks hanging on the same wall. The observation made by Huygens launched research into coupled oscillator systems. From the viewpoint of collective behavior science is interested in the emerging synchronization of a large number of coupled oscillators. In a broad sense any system exhibiting a quasi-periodic dynamics can be considered an oscillator. Such systems are very frequent in nature, and the most interesting ones are in living organisms. Pacemaker cells in the heart or neurons which control rhythmic activities are capable of synchronization. Synchronization of chirping crickets, flashing fireflies in south-east Asia, menstrual cycles in women or clapping of spectators are also well-known examples [Pikovsky *et al.*, 2002]. Synchronization in such systems have a more complex mechanism than synchronization of simple physical pendulums. Biological and sociological systems usually do have a tendency to optimize their evolution, and probably synchronization is not their primary aim, it is just a byproduct of some complex optimization procedure. Models that aim to describe realistically such systems should take this difference into consideration and should consider a more complex approach than those offered by interacting mechanical oscillators. The present paper intends to contribute in such sense by studying theoretically and experimentally the collective behavior of a system of flashing oscillators governed by a simple optimization dynamics. More specifically, we will revisit the simple physical system described in our recent works [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009] and extend it by considering a novel dynamics and an experimental realization for this.

Despite the fact that Huygens discovered the synchronization in the 17th century, mathematical models appeared just after 1960, and were elaborated by biologists. The models progressed rapidly due to the quick evolution of the computers, and by studying methods borrowed from physics and mathematics (for a review please consult [Strogatz, 2000]). Most of these models fall into two broad categories: those describing phase-coupled oscillators, and those which use pulse-coupling between the units.

The classical phase-coupled model of synchronization was first introduced by Winfree [1967] and solved analytically by Kuramoto and Nishikawa [1987]. These type of models are known as Kuramoto type models. In the simplest version of the model, each oscillator has an associated phase between 0 and 2π. The oscillators evolve according to a set of coupled first order differential equations, with a coupling that minimizes the phase difference between them. The form of the coupling was chosen by Kuramoto and Nishikawa in a manner that allows for an analytic solution. In the thermodynamic limit, this model shows a second-order phase transition as a function of the coupling strength. There is a critical coupling and below this the system does not synchronize. Above the critical coupling value the system exhibits partial synchronization, and the synchronization level increases in a monotonic manner with the coupling strength. The critical coupling depends on the variance of the oscillators' frequencies. An ensemble of oscillators with widely different frequencies will have a large critical coupling value, whereas a system composed by oscillators with similar frequencies will have a low critical coupling.

Pulse-coupled oscillators are mainly used in integrate-and-fire type models [Burkitt, 2006]. These models are probably the simplest ones able to approximate the collective behavior of a neuron ensemble [FitzHugh, 1955]. Each oscillator has a phase and a state variable, linked by a monotonic function. The dynamics of an isolated oscillator is simple: the phase increases monotonically until it reaches a given value. At this point the oscillator emits a pulse (it *fires*) and resets its state and phase-variable. In an ensemble of interacting oscillators when one oscillator fires, the state variable of all those other oscillators that can detect the emitted pulse increases instantly by a fixed value. Accordingly, the phase of these oscillators increases too. Under the right conditions, a single pulse can trigger an avalanche of pulses in the system causing a high proportion of the units to fire at the same time. Synchronization in such systems can appear under very broad conditions. Detailed studies concerning synchronization in such systems were done in [Mirollo & Strogatz, 1990; Bottani, 1996].

Synchronization of firing oscillators acquired new perspectives and interest with the pioneering work of Wolf Singer [Singer, 2001], after his hypothesis of the mammalian brain that generates continuously highly dynamic states which are modulated by input signals. Due to this modulation the system of neurons rapidly converge towards points of transient stability that correspond to the respective input constellation.

In the present work we consider a different type of synchronization mechanism: one in which the interaction between the oscillating units is not chosen explicitly to induce synchronization. Instead of being the result of an evident phase-difference minimizing force, synchronization arises as a side effect of a simple optimization rule. Such models are described in our earlier works [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009]. The present work aims to extend the conditions under which synchronization can be obtained by following a simple optimization dynamics. Such systems are interesting in the view of the hypothesis formulated by Singer.

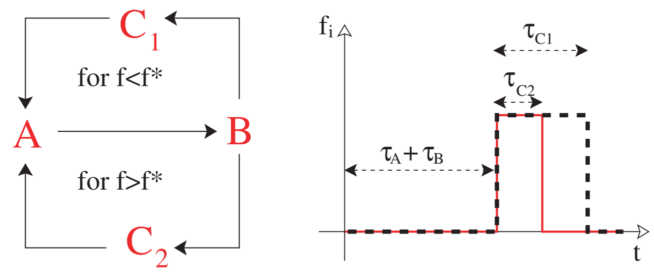
2. The Model

The synchronization model under investigation consists of an assembly of N oscillators able to emit and detect pulses. At each time-moment the oscillators can either be active by outputting a signal (pulse) of strength $1/N$, or inactive, outputting no signal at all. The total output intensity of the assembly will vary thus between 0 and 1. The easiest is to picture these oscillators as flashing units (this is in fact how the experimental setup works), so the active oscillators will be referred to as *lit*, while the inactive ones will be called *unlit*. Correspondingly, the total output level of the system can be thought of as the total light intensity. The units are stochastic oscillators, which means that their period fluctuates in time. They are also multimode elements which means that as a function of the global signal in the system they can operate in different modes. In previous studies, the possible modes were distinguished by the time-length of the inactive states. In contrast to this, here we investigate systems where the modes are distinguished by the time-length of the active (light emitting) state.

Each oscillator cycles between three states, which will be denoted here by A, B and C [see Fig. 1(a)] [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009]. We discuss in detail now the model, following the A, B and C states of the cycle. State A is the stochastic part of the oscillators period. Its length, τ_A, is a random variable following an exponential distribution:

$$\rho(\tau_A) = \frac{1}{\tau^*} e^{-(\tau_A/\tau^*)}. \tag{1}$$

The average duration of state A is $\langle \tau_A \rangle = \tau^*$. This distribution is the one that describes the escape times of a stochastically driven particle from



(a) (b)

Fig. 1. (a) Sketch of the oscillator dynamics. Each oscillator cycles between three states, A, B and C. As a function of the f/f^* value two modes are possible. (b) Output of oscillator i as a function of time. The duration of phase C can be long or short (τ_{C1} or τ_{C2}).

a potential valley, and it is believed to be relevant for neuronal systems as well. In fact, one can choose any normalized distribution for the τ_A values, without influencing the main results of the present work.

State B is the waiting or "charging time" and has a fixed length τ_B. This state ensures that the oscillator stays unlit for at least a time-length τ_B. In the previous studies, the oscillation modes were distinguished by the length of this state. This means, that several discrete time-lengths were allowed for τ_B. In the simplest version of the model τ_B could take two possible values: τ_{B1} and $\tau_{B2} = 2\tau_{B1}$. In the model considered here the time-length of state B is the same for all possible modes.

State C is the lit state of the units, and in the present model can have different durations. The duration of state C will distinguish between the modes. We consider two possible values for the duration of state C, a longer one, τ_{C1}, and a shorter one, τ_{C2}. These correspond to oscillating modes 1 and 2, respectively (see Fig. 1).

The units follow an output intensity optimization dynamics, similar to the one used in the previous studies [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009]. When entering state C, the oscillators decide how long to stay in that state based on the total output of the system. If the total output f is less than a prescribed value f^*, the oscillator will choose mode 1. If $f > f^*$, it will operate in mode 2.

By choosing mode 1 when $f < f^*$ and choosing mode 2 when $f > f^*$, the units try to keep the output close to f^*. In such manner the dynamics optimizes the value of the total output by changing the length of the lit period. The system is globally coupled, since each unit detects the output of all the other units. In the earlier models [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009], instead of phase C, it was the length of phase B that was different between the two modes. Note that unlike in the earlier versions, in the present model the length of a full oscillation period is lengthened rather than shortened by a low total output. As a side effect of this optimization, for certain values of the model parameters the oscillators will flash synchronously, and the total output of the system becomes approximately periodic in time. This is the collective behavior we are interested in.

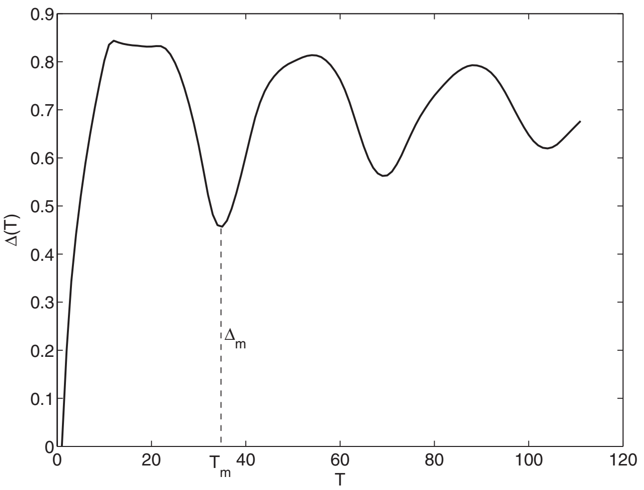


Fig. 2. Shape of the $\Delta(T)$ function for parameters $f^* = 0.2$, $\tau^* = 0.05$ and $N = 2000$.

3. The Periodicity Level of the Output

To back up the claim that the units are capable of flashing in unison, we need an objective way to detect and measure synchronization in the system. Several order parameters are appropriate for this, however most of them needs the knowledge of the states for all the individual units. Here a simple order parameter will be used, that can be determined solely from the knowledge of the total output intensity as a function of time: $f(t)$. This order parameter characterizes the periodicity level of the global signal, and it is suitable for those experimental studies where one has knowledge only about the total output intensity in the system.

For calculating the order parameter, first a $\Delta(T)$ function is defined. This carries information about how appropriate a periodic function with period T is to characterize the global signal $f(t)$ ([Nikitin *et al.*, 2001; Néda *et al.*, 2003]):

$$\Delta(T) = \frac{1}{2M} \lim_{x\to\infty} \frac{1}{x} \int_0^x |f(t) - f(T)| dt \qquad (2)$$

where

$$M = \lim_{x\to\infty} \frac{1}{x} \int_0^x |f(t) - \langle f(t)\rangle| dt \quad \text{and}$$
$$\langle f(t)\rangle = \lim_{x\to\infty} \frac{1}{x} \int_0^x f(t) dt \qquad (3)$$

The general shape of this function is sketched in Fig. 2. The more periodic the output signal is (assuming a period T), the smaller the value of $\Delta(T)$ will be. Thus the period of the approximately periodic function $f(t)$ can be considered to be T_m, where $\Delta_m = \Delta(T_m)$ is the deepest minimum of $\Delta(T)$ (excluding the obvious minimum at $T = 0$). It is easy to see that for a perfectly periodic function of period T_1, $\Delta_m(T_1) = 0$.

We define now the *periodicity level* of the output as $p = 1/\Delta_m$. The quantity p/p_1, was chosen as the order parameter characterizing the synchronization level of the assembly of oscillators. Here, p_1 is the periodicity level of a single unit oscillating in the long mode (C_1), hence p/p_1 is the increase in the periodicity level of the output due to the coupling between the oscillators.

4. Simulation Results

First the model is studied by computer simulations. Following our earlier studies we were primarily interested about the influence of the main parameters τ^* and f^* on the synchronization level. The other parameters of the model were fixed at the following values: $\tau_B = 0.8$, $\tau_{C1} = 0.4$ and $\tau_{C2} = 0.2$ units. Considering other choices for these values leads to qualitatively similar results.

Similarly to earlier models and studies, where the modes differed in the value of τ_B, the present system partially synchronizes for an island-like region in the τ^*–f^* parameter space (Fig. 3). For a given τ^* value, synchronization starts at a $f^*_{\min} > 0$ value and disappears over an $f^*_{\max}$ value. As the value of τ^* increases the f^* interval in which synchronization is present gets shorter and the $f^*_{\min}$ values get also smaller.

It was found that in the region where synchronization is present, the synchronization level quantified by p/p_1 increases monotonically with the

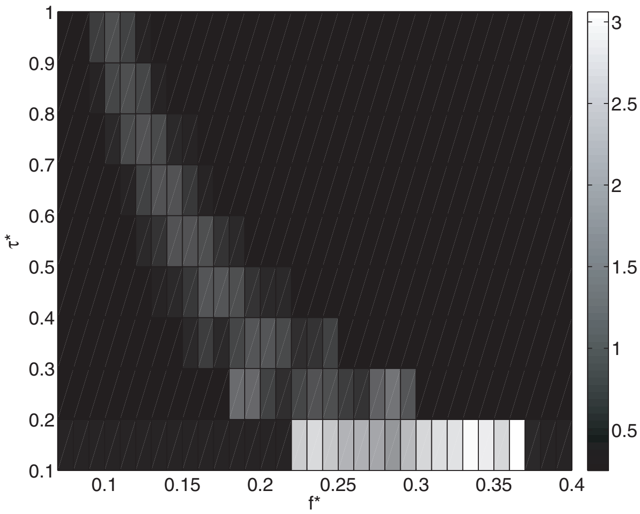


Fig. 3. Synchronization level as a function of τ^* and f^*. Lighter shades of grey indicate a higher p/p_1 value. Synchronization occurs only in a certain region of the f^*-τ^* space.

number of units in the system (see Fig. 4). Moreover, the $p/p_1 > 1$ values suggest that the ensemble as a whole generates a signal with better periodicity than the isolated units by their self. These results are in agreement with the findings in the earlier models, where the value of τ_B distinguished the two modes.

For a fixed τ^* value, synchronization appears and disappears abruptly as f^* is varied [see Fig. 5(a)]. It is also observable that the transition gets sharper as the number of units, N, is increased, suggesting dynamic phase-transitions in the system.

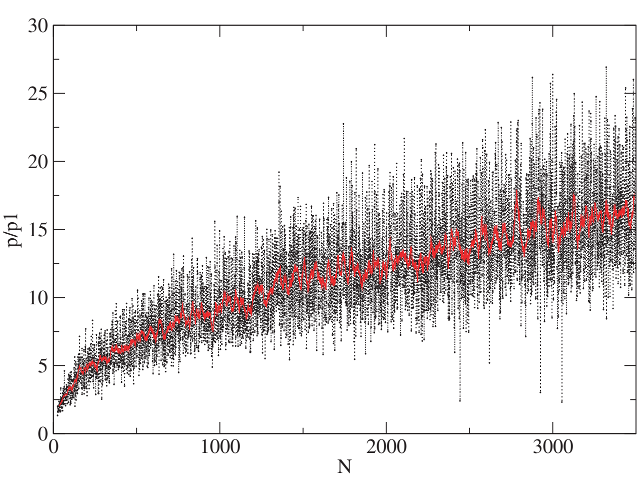
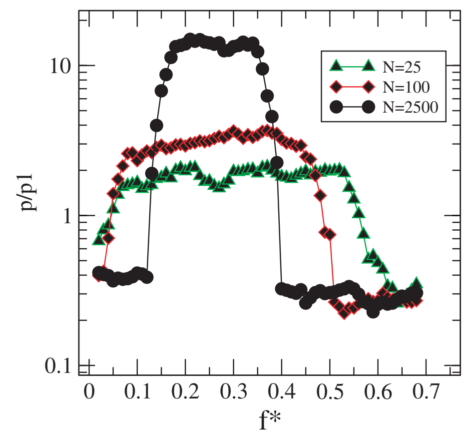
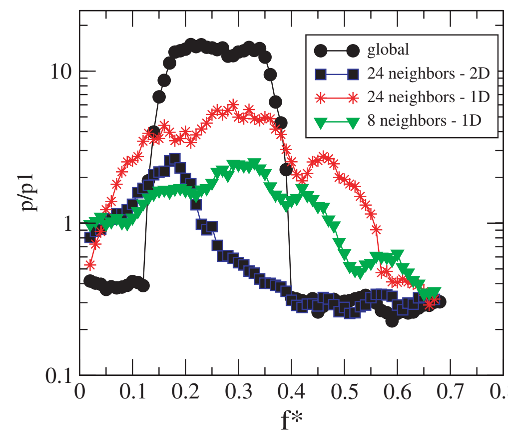


Fig. 4. Synchronization level as a function of the number of oscillators ($\tau^* = 0.05$ and $f^* = 0.3$). Small black dots connected by thin dashed line indicates the rough simulation results and the red line indicates an average realized on a moving window of length $\Delta N = 50$.



(a)



(b)

Fig. 5. Synchronization level as a function of f^*. (a) Results for the case of global coupling (τ^* was fixed at 0.05, and the number of units, N, is indicated in the legend). (b) Comparison of results for different types of coupling: global, coupling with neighbors in 1D and 2D geometry for a system composed of $N = 2500$ oscillators.

This nonequilibrium phase transition seems quite different from the ones discussed in the literature [Henkel *et al.*, 2008]. The system has no absorbing states, and synchronization is achieved by persistent shift between the modes without a phase-locking mechanism. The phase-transition in this system is thus a special one, and one cannot simply categorize it in the presently established classes. It is worth mentioning that synchronization appears also in the case of local coupling, when the oscillator detects only the pulses emitted by its neighbors.

Considering a lattice topology and interaction with enough neighbors, the synchronization scenario as a function of f^* is similar. In Fig. 5(b), we show results in this sense considering $N = 2500$

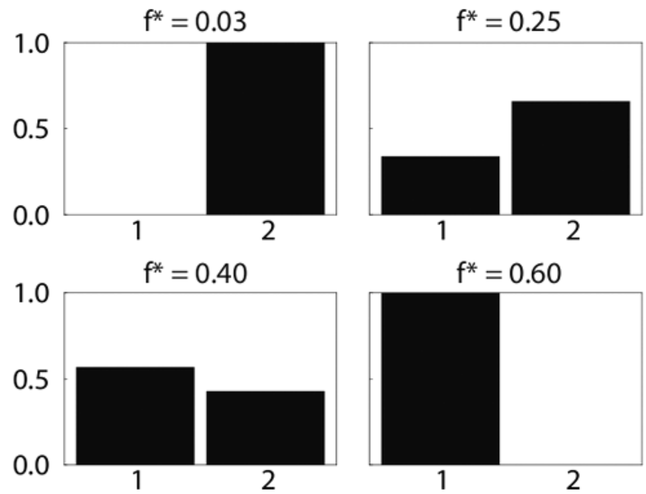


Fig. 6. Distribution of the oscillation modes for various values of f^*. Results obtained for $\tau^* = 0.05$ and $N = 2000$.

oscillators placed on a one-dimensional (1D) lattice or on a two-dimensional (2D) square-lattice. We have observed that when considering only interactions with a few neighbors (in 1D 2–4 and in 2D 4–8), synchronization is not detectable.

Similarly with the model discussed in [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009] synchronization appears when both modes are active. This means that the units continuously shift between the modes in order to optimize the average output level around the desired f^* value. In order to prove this, the distribution of the followed oscillation modes is plotted in Fig. 6 for a few values of f^*. As expected, for those values of f^* where synchronization occurs, both modes are present. For low or high f^* values solely one mode occurs. There are only random shifts in the length of oscillation periods and thus synchronization cannot appear.

5. Experimental Realization

We consider now an experimental realization of the discussed two-mode oscillator ensemble. Electronic oscillators capable of light emission and detection were built. Due to their flashing behavior we called these units "electronic fireflies". These electronic fireflies are integrated circuits having a simple circuit diagram (Fig. 7).

The heart of the oscillators is an 8 bit RISC-core (Reduced Instruction Set Computer) microcontroller from Atmel. Figure 8 presents a picture of one unit, where one can easily identify the main parts of the electronic firefly: the micro-controller, the photo-resistor, and the Light Emitting Diode (LED). To ensure a natural behavior, as time reference the internal RC oscillator of the microcontroller was chosen. The photo-resistor measures the light intensity in the system, and it is in conjunction with three normal resistors of 10 KΩ, 100 KΩ, and 1 MΩ, which enables several sensitivity ranges. Setting up a low pass filter on the photoresistor signal by choosing one of the two 10 nF or 100 nF capacitors on the INT0 and INT1 pins is also possible.

The reference signal U (which corresponds to the value of f^* in the model) applied to the oscillators and the voltage on the photo-resistor can

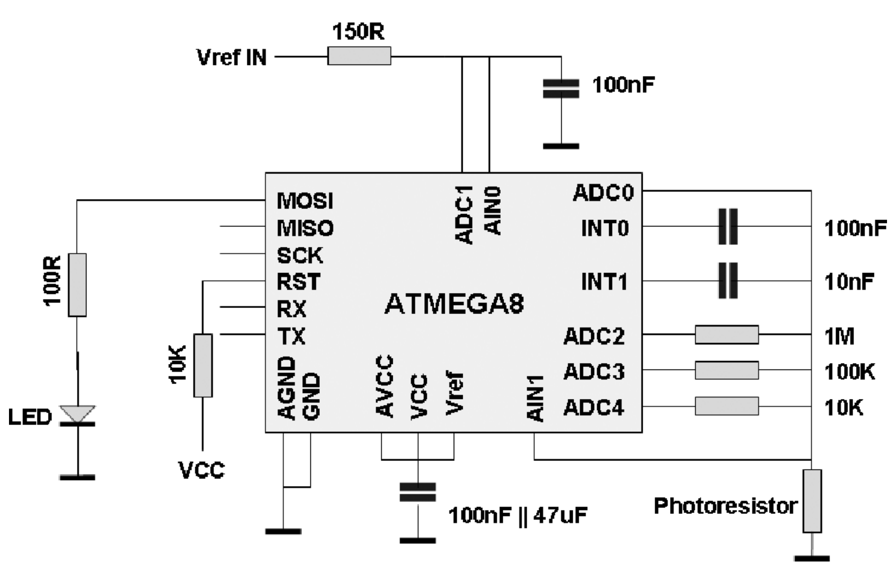


Fig. 7. Circuit diagram of the flashing multimode oscillators.

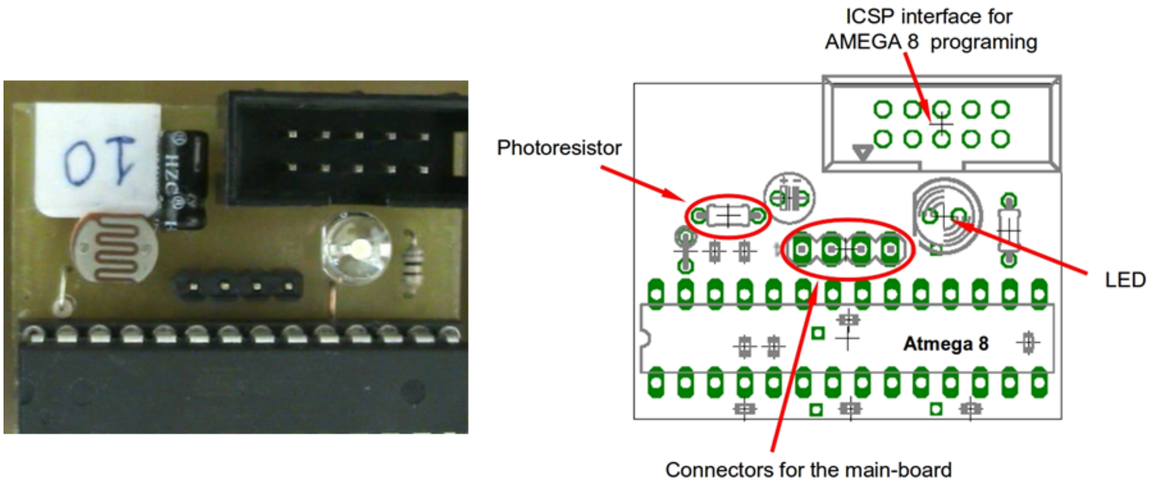


Fig. 8. Picture of the flashing oscillators (electronic fireflies) and sketch of the main electronic parts.

either be measured by a 10-bit resolution analog-digital converter or compared by a built-in hardware comparator. The output of the oscillator will be given by the LED. A hardware pulse width modulation can be used to alter this light intensity. The main advantage of this system is the software-oriented flexibility. Once the τ_B, τ_{C1} and τ_{C2} parameters are fixed we burn the program in the micro-controller's Programmable Erasable Read-Only Memory (EPROM). Due to the fact, that we use real electronic components, there is an intrinsically built in stochasticity in the system, so we do not need an extra stochastic state. This small stochasticity is equivalent with nonzero, but still very small τ^* value. The program stored in the EPROM governs the dynamic of the units. The electronic fireflies are placed on a circuit board connected to a PC, which drives the input and output of the relevant information like in which state the oscillators are and what are the values of f^*. In Fig. 9 we present this circuit board and its PC interface.

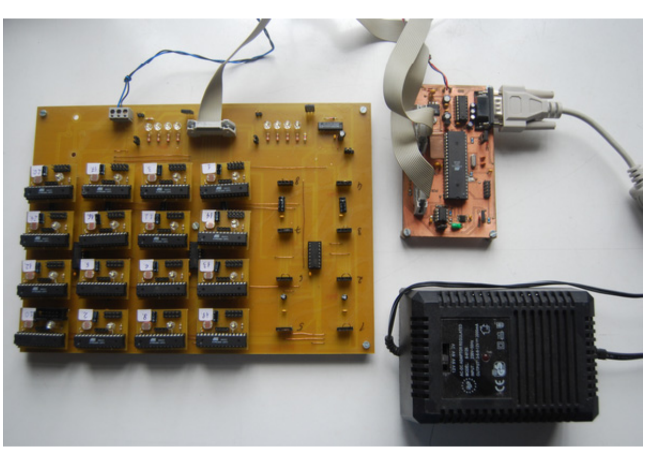


Fig. 9. Circuit board with the electronic fireflies and the computer interface.

In order to assure, that the system is globally coupled and is not influenced from the outside light, we isolate the system in a box covered with tinfoil, which reflects, and scatters the light emitted by the electronic fireflies, so the coupling in the system is global with a fair approximation. A program running on the PC helps to gather information about the state of the units, saving all the relevant information in a file. This program allows also to change the value of the reference voltage U (which corresponds to the value of f^*) and to control the length of the measurement times.

6. Experimental Results

Experiments were carried out using a relatively small system of 4×4 flashing two-mode oscillators. The oscillators were programmed with the $\tau_B = 768\,\text{ms}$, $\tau_{C1} = 384\,\text{ms}$ and $\tau_{C2} = 192\,\text{ms}$ values. These values are chosen proportionally with the ones used in computer simulations. The U threshold parameter, which corresponds to f^*, was varied between 0 and 5000 mV, with a step of 10 mV. For each U value data collection for 10 minutes were done. Recordings for the first two minutes were neglected since it was considered as a transient period. The p order parameter was computed in the remaining 8 minutes. Results are plotted in Fig. 10.

The experimental parameters were chosen proportionally with those ones in computer

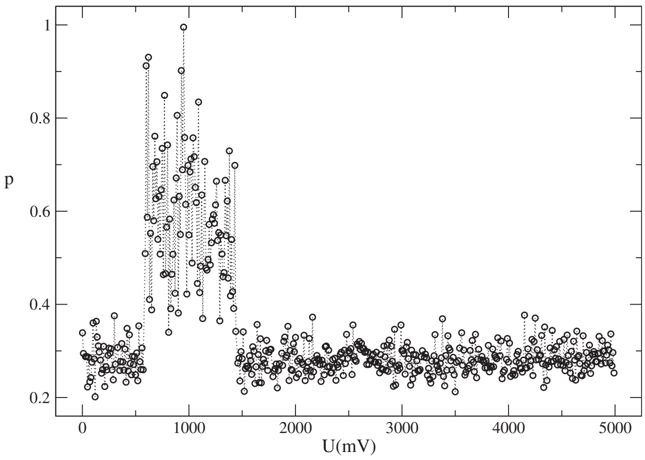


Fig. 10. Experimental results for a system of 4×4 oscillators. The periodicity level p is plotted as a function of the U threshold value. The system parameters are: $\tau_B = 768\,\text{ms}$, $\tau_{C1} = 384\,\text{ms}$ and $\tau_{C2} = 192\,\text{ms}$.

simulations. One can immediately observe that experiments yield similar results with computer simulations of the model. Synchronization, as characterized by the p periodicity level, appears for a finite interval of the U threshold value, similarly with the behavior depicted in Fig. 5. Unfortunately due to the small number of available flashing oscillators and the small number of available places on the common circuit board we could not study larger systems. For much smaller systems however (4–9 units), synchronization did not appear, so in such conditions finite-size effects were not possible to investigate experimentally.

7. Conclusions

An ensemble of simple oscillators capable of complex emerging behavior was studied by computer simulations and by a simple experimental setup. The considered oscillators are multimode stochastic elements, resembling those considered in earlier studies [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009]. In each period the oscillators emit a pulse with a finite duration and detect the pulses emitted by the others. The modes are distinguished by the length of the emitted pulses. The system is different from that already studied in the literature, where the modes are distinguished by the length of the waiting-time between two consecutive pulses. The dynamics of the units is governed by the same output-intensity optimization dynamics as the one considered in [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009]: whenever the total output detected by one oscillator is lower than an f^* threshold value, the oscillator follows a mode with a longer pulse, and when the detected total output is higher than the f^* value it follows a mode with a shorter pulse-length. The aim of the present research was to investigate whether the output-intensity optimization realized by the varying pulse-length will be able to reproduce the previously observed nontrivial synchronization and periodicity enhancement. The main conclusion is that similarly with the results obtained in [Nikitin *et al.*, 2001; Néda *et al.*, 2003; Sumi *et al.*, 2009] partial synchronization appears for a given f^* threshold interval. Increasing the number of oscillators in the system will increase the periodicity level of the global output. Surprisingly, the periodicity level of the global signal can be higher than the periodicity of one isolated unit, thus starting from nonperfect oscillators the collective dynamics produce an oscillator with stable period. This is a clear fault-tolerant behavior, which allows to engineer oscillators with very stable periods using imperfect units. The periodicity of the system is also stable against faults in the individual oscillators. We believe that biological systems might use similar mechanism for achieving a stable circadian rhythm. Following the novel ideas formulated by Singer [2001], such systems could be also useful for detection purposes, since the collective behavior is rather sensible for the threshold value. Further studies can consider thus the onset of collective behavior depending on spatial and/or temporal patterns.

Acknowledgments

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